

## B Integration Tables

### Forms Involving $u^n$

$$1. \int u^n du = \frac{u^{n+1}}{n+1} + C, \quad n \neq -1$$

$$2. \int \frac{1}{u} du = \ln|u| + C$$

### Forms Involving $a + bu$

$$3. \int \frac{u}{a+bu} du = \frac{1}{b^2}(bu - a \ln|a+bu|) + C$$

$$4. \int \frac{u}{(a+bu)^2} du = \frac{1}{b^2}\left(\frac{a}{a+bu} + \ln|a+bu|\right) + C$$

$$5. \int \frac{u}{(a+bu)^n} du = \frac{1}{b^2}\left[\frac{-1}{(n-2)(a+bu)^{n-2}} + \frac{a}{(n-1)(a+bu)^{n-1}}\right] + C, \quad n \neq 1, 2$$

$$6. \int \frac{u^2}{a+bu} du = \frac{1}{b^3}\left[-\frac{bu}{2}(2a-bu) + a^2 \ln|a+bu|\right] + C$$

$$7. \int \frac{u^2}{(a+bu)^2} du = \frac{1}{b^3}\left(bu - \frac{a^2}{a+bu} - 2a \ln|a+bu|\right) + C$$

$$8. \int \frac{u^2}{(a+bu)^3} du = \frac{1}{b^3}\left[\frac{2a}{a+bu} - \frac{a^2}{2(a+bu)^2} + \ln|a+bu|\right] + C$$

$$9. \int \frac{u^2}{(a+bu)^n} du = \frac{1}{b^3}\left[\frac{-1}{(n-3)(a+bu)^{n-3}} + \frac{2a}{(n-2)(a+bu)^{n-2}} - \frac{a^2}{(n-1)(a+bu)^{n-1}}\right] + C, \quad n \neq 1, 2, 3$$

$$10. \int \frac{1}{u(a+bu)} du = \frac{1}{a} \ln\left|\frac{u}{a+bu}\right| + C$$

$$11. \int \frac{1}{u(a+bu)^2} du = \frac{1}{a}\left(\frac{1}{a+bu} + \frac{1}{a} \ln\left|\frac{u}{a+bu}\right|\right) + C$$

$$12. \int \frac{1}{u^2(a+bu)} du = -\frac{1}{a}\left(\frac{1}{u} + \frac{b}{a} \ln\left|\frac{u}{a+bu}\right|\right) + C$$

$$13. \int \frac{1}{u^2(a+bu)^2} du = -\frac{1}{a^2}\left[\frac{a+2bu}{u(a+bu)} + \frac{2b}{a} \ln\left|\frac{u}{a+bu}\right|\right] + C$$

### Forms Involving $a + bu + cu^2, b^2 \neq 4ac$

$$14. \int \frac{1}{a+bu+cu^2} du = \begin{cases} \frac{2}{\sqrt{4ac-b^2}} \arctan \frac{2cu+b}{\sqrt{4ac-b^2}} + C, & b^2 < 4ac \\ \frac{1}{\sqrt{b^2-4ac}} \ln \left| \frac{2cu+b-\sqrt{b^2-4ac}}{2cu+b+\sqrt{b^2-4ac}} \right| + C, & b^2 > 4ac \end{cases}$$

$$15. \int \frac{u}{a+bu+cu^2} du = \frac{1}{2c} \left( \ln|a+bu+cu^2| - b \int \frac{1}{a+bu+cu^2} du \right)$$

### Forms Involving $\sqrt{a+bu}$

$$16. \int u^n \sqrt{a+bu} du = \frac{2}{b(2n+3)} \left[ u^n(a+bu)^{3/2} - na \int u^{n-1} \sqrt{a+bu} du \right]$$

$$17. \int \frac{1}{u\sqrt{a+bu}} du = \begin{cases} \frac{1}{\sqrt{a}} \ln \left| \frac{\sqrt{a+bu} - \sqrt{a}}{\sqrt{a+bu} + \sqrt{a}} \right| + C, & a > 0 \\ \frac{2}{\sqrt{-a}} \arctan \sqrt{\frac{a+bu}{-a}} + C, & a < 0 \end{cases}$$

$$18. \int \frac{1}{u^n \sqrt{a+bu}} du = \frac{-1}{a(n-1)} \left[ \frac{\sqrt{a+bu}}{u^{n-1}} + \frac{(2n-3)b}{2} \int \frac{1}{u^{n-1} \sqrt{a+bu}} du \right], \quad n \neq 1$$

19.  $\int \frac{\sqrt{a+bu}}{u} du = 2\sqrt{a+bu} + a \int \frac{1}{u\sqrt{a+bu}} du$
20.  $\int \frac{\sqrt{a+bu}}{u^n} du = \frac{-1}{a(n-1)} \left[ \frac{(a+bu)^{3/2}}{u^{n-1}} + \frac{(2n-5)b}{2} \int \frac{\sqrt{a+bu}}{u^{n-1}} du \right], n \neq 1$
21.  $\int \frac{u}{\sqrt{a+bu}} du = \frac{-2(2a-bu)}{3b^2} \sqrt{a+bu} + C$
22.  $\int \frac{u^n}{\sqrt{a+bu}} du = \frac{2}{(2n+1)b} \left( u^n \sqrt{a+bu} - na \int \frac{u^{n-1}}{\sqrt{a+bu}} du \right)$

Forms Involving  $a^2 \pm u^2, a > 0$

23.  $\int \frac{1}{a^2 + u^2} du = \frac{1}{a} \arctan \frac{u}{a} + C$
24.  $\int \frac{1}{u^2 - a^2} du = - \int \frac{1}{a^2 - u^2} du = \frac{1}{2a} \ln \left| \frac{u-a}{u+a} \right| + C$
25.  $\int \frac{1}{(a^2 \pm u^2)^n} du = \frac{1}{2a^2(n-1)} \left[ \frac{u}{(a^2 \pm u^2)^{n-1}} + (2n-3) \int \frac{1}{(a^2 \pm u^2)^{n-1}} du \right], n \neq 1$

Forms Involving  $\sqrt{u^2 \pm a^2}, a > 0$

26.  $\int \sqrt{u^2 \pm a^2} du = \frac{1}{2} (u\sqrt{u^2 \pm a^2} \pm a^2 \ln|u + \sqrt{u^2 \pm a^2}|) + C$
27.  $\int u^2 \sqrt{u^2 \pm a^2} du = \frac{1}{8} [u(2u^2 \pm a^2) \sqrt{u^2 \pm a^2} - a^4 \ln|u + \sqrt{u^2 \pm a^2}|] + C$
28.  $\int \frac{\sqrt{u^2 + a^2}}{u} du = \sqrt{u^2 + a^2} - a \ln \left| \frac{a + \sqrt{u^2 + a^2}}{u} \right| + C$
29.  $\int \frac{\sqrt{u^2 - a^2}}{u} du = \sqrt{u^2 - a^2} - a \operatorname{arcsec} \frac{|u|}{a} + C$
30.  $\int \frac{\sqrt{u^2 \pm a^2}}{u^2} du = \frac{-\sqrt{u^2 \pm a^2}}{u} + \ln|u + \sqrt{u^2 \pm a^2}| + C$
31.  $\int \frac{1}{\sqrt{u^2 \pm a^2}} du = \ln|u + \sqrt{u^2 \pm a^2}| + C$
32.  $\int \frac{1}{u\sqrt{u^2 + a^2}} du = \frac{-1}{a} \ln \left| \frac{a + \sqrt{u^2 + a^2}}{u} \right| + C$
33.  $\int \frac{1}{u\sqrt{u^2 - a^2}} du = \frac{1}{a} \operatorname{arcsec} \frac{|u|}{a} + C$
34.  $\int \frac{u^2}{\sqrt{u^2 \pm a^2}} du = \frac{1}{2} (u\sqrt{u^2 \pm a^2} \mp a^2 \ln|u + \sqrt{u^2 \pm a^2}|) + C$
35.  $\int \frac{1}{u^2 \sqrt{u^2 \pm a^2}} du = \mp \frac{\sqrt{u^2 \pm a^2}}{a^2 u} + C$
36.  $\int \frac{1}{(u^2 \pm a^2)^{3/2}} du = \frac{\pm u}{a^2 \sqrt{u^2 \pm a^2}} + C$

Forms Involving  $\sqrt{a^2 - u^2}, a > 0$

37.  $\int \sqrt{a^2 - u^2} du = \frac{1}{2} \left( u\sqrt{a^2 - u^2} + a^2 \arcsin \frac{u}{a} \right) + C$
38.  $\int u^2 \sqrt{a^2 - u^2} du = \frac{1}{8} \left[ u(2u^2 - a^2) \sqrt{a^2 - u^2} + a^4 \arcsin \frac{u}{a} \right] + C$

$$39. \int \frac{\sqrt{a^2 - u^2}}{u} du = \sqrt{a^2 - u^2} - a \ln \left| \frac{a + \sqrt{a^2 - u^2}}{u} \right| + C$$

$$40. \int \frac{\sqrt{a^2 - u^2}}{u^2} du = \frac{-\sqrt{a^2 - u^2}}{u} - \arcsin \frac{u}{a} + C$$

$$41. \int \frac{1}{\sqrt{a^2 - u^2}} du = \arcsin \frac{u}{a} + C$$

$$42. \int \frac{1}{u\sqrt{a^2 - u^2}} du = \frac{-1}{a} \ln \left| \frac{a + \sqrt{a^2 - u^2}}{u} \right| + C$$

$$43. \int \frac{u^2}{\sqrt{a^2 - u^2}} du = \frac{1}{2} \left( -u\sqrt{a^2 - u^2} + a^2 \arcsin \frac{u}{a} \right) + C$$

$$44. \int \frac{1}{u^2\sqrt{a^2 - u^2}} du = \frac{-\sqrt{a^2 - u^2}}{a^2 u} + C$$

$$45. \int \frac{1}{(a^2 - u^2)^{3/2}} du = \frac{u}{a^2\sqrt{a^2 - u^2}} + C$$

Forms Involving  $\sin u$  or  $\cos u$ 

$$46. \int \sin u du = -\cos u + C$$

$$47. \int \cos u du = \sin u + C$$

$$48. \int \sin^2 u du = \frac{1}{2}(u - \sin u \cos u) + C$$

$$49. \int \cos^2 u du = \frac{1}{2}(u + \sin u \cos u) + C$$

$$50. \int \sin^n u du = -\frac{\sin^{n-1} u \cos u}{n} + \frac{n-1}{n} \int \sin^{n-2} u du$$

$$51. \int \cos^n u du = \frac{\cos^{n-1} u \sin u}{n} + \frac{n-1}{n} \int \cos^{n-2} u du$$

$$52. \int u \sin u du = \sin u - u \cos u + C$$

$$53. \int u \cos u du = \cos u + u \sin u + C$$

$$54. \int u^n \sin u du = -u^n \cos u + n \int u^{n-1} \cos u du$$

$$55. \int u^n \cos u du = u^n \sin u - n \int u^{n-1} \sin u du$$

$$56. \int \frac{1}{1 \pm \sin u} du = \tan u \mp \sec u + C$$

$$57. \int \frac{1}{1 \pm \cos u} du = -\cot u \pm \csc u + C$$

$$58. \int \frac{1}{\sin u \cos u} du = \ln |\tan u| + C$$

Forms Involving  $\tan u$ ,  $\cot u$ ,  $\sec u$ , or  $\csc u$ 

$$59. \int \tan u du = -\ln |\cos u| + C$$

$$60. \int \cot u du = \ln |\sin u| + C$$

$$61. \int \sec u du = \ln |\sec u + \tan u| + C$$

$$62. \int \csc u du = \ln |\csc u - \cot u| + C \quad \text{or} \quad \int \csc u du = -\ln |\csc u + \cot u| + C$$

$$63. \int \tan^2 u du = -u + \tan u + C$$

$$64. \int \cot^2 u du = -u - \cot u + C$$

$$65. \int \sec^2 u du = \tan u + C$$

$$66. \int \csc^2 u du = -\cot u + C$$

$$67. \int \tan^n u du = \frac{\tan^{n-1} u}{n-1} - \int \tan^{n-2} u du, \quad n \neq 1$$

$$68. \int \cot^n u du = -\frac{\cot^{n-1} u}{n-1} - \int (\cot^{n-2} u) du, \quad n \neq 1$$

$$69. \int \sec^n u du = \frac{\sec^{n-2} u \tan u}{n-1} + \frac{n-2}{n-1} \int \sec^{n-2} u du, \quad n \neq 1$$

$$70. \int \csc^n u du = -\frac{\csc^{n-2} u \cot u}{n-1} + \frac{n-2}{n-1} \int \csc^{n-2} u du, \quad n \neq 1$$

**A6 Appendix B** Integration Tables

$$71. \int \frac{1}{1 \pm \tan u} du = \frac{1}{2}(u \pm \ln|\cos u \pm \sin u|) + C$$

$$73. \int \frac{1}{1 \pm \sec u} du = u + \cot u \mp \csc u + C$$

$$72. \int \frac{1}{1 \pm \cot u} du = \frac{1}{2}(u \mp \ln|\sin u \pm \cos u|) + C$$

$$74. \int \frac{1}{1 \pm \csc u} du = u - \tan u \pm \sec u + C$$

**Forms Involving Inverse Trigonometric Functions**

$$75. \int \arcsin u du = u \arcsin u + \sqrt{1-u^2} + C$$

$$77. \int \arctan u du = u \arctan u - \ln\sqrt{1+u^2} + C$$

$$79. \int \operatorname{arcsec} u du = u \operatorname{arcsec} u - \ln|u + \sqrt{u^2-1}| + C$$

$$76. \int \arccos u du = u \arccos u - \sqrt{1-u^2} + C$$

$$78. \int \operatorname{arccot} u du = u \operatorname{arccot} u + \ln\sqrt{1+u^2} + C$$

$$80. \int \operatorname{arccsc} u du = u \operatorname{arccsc} u + \ln|u + \sqrt{u^2-1}| + C$$

**Forms Involving  $e^u$**

$$81. \int e^u du = e^u + C$$

$$83. \int u^n e^u du = u^n e^u - n \int u^{n-1} e^u du$$

$$85. \int e^{au} \sin bu du = \frac{e^{au}}{a^2 + b^2}(a \sin bu - b \cos bu) + C$$

$$82. \int u e^u du = (u-1)e^u + C$$

$$84. \int \frac{1}{1+e^u} du = u - \ln(1+e^u) + C$$

$$86. \int e^{au} \cos bu du = \frac{e^{au}}{a^2 + b^2}(a \cos bu + b \sin bu) + C$$

**Forms Involving  $\ln u$**

$$87. \int \ln u du = u(-1 + \ln u) + C$$

$$89. \int u^n \ln u du = \frac{u^{n+1}}{(n+1)^2}[-1 + (n+1)\ln u] + C, n \neq -1$$

$$90. \int (\ln u)^2 du = u[2 - 2\ln u + (\ln u)^2] + C$$

$$88. \int u \ln u du = \frac{u^2}{4}(-1 + 2\ln u) + C$$

$$91. \int (\ln u)^n du = u(\ln u)^n - n \int (\ln u)^{n-1} du$$

**Forms Involving Hyperbolic Functions**

$$92. \int \cosh u du = \sinh u + C$$

$$94. \int \operatorname{sech}^2 u du = \tanh u + C$$

$$96. \int \operatorname{sech} u \tanh u du = -\operatorname{sech} u + C$$

$$93. \int \sinh u du = \cosh u + C$$

$$95. \int \operatorname{csch}^2 u du = -\operatorname{coth} u + C$$

$$97. \int \operatorname{csch} u \operatorname{coth} u du = -\operatorname{csch} u + C$$

**Forms Involving Inverse Hyperbolic Functions (in logarithmic form)**

$$98. \int \frac{du}{\sqrt{u^2 \pm a^2}} = \ln(u + \sqrt{u^2 \pm a^2}) + C$$

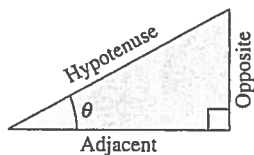
$$100. \int \frac{du}{u\sqrt{a^2 \pm u^2}} = -\frac{1}{a} \ln \frac{a + \sqrt{a^2 \pm u^2}}{|u|} + C$$

$$99. \int \frac{du}{a^2 - u^2} = \frac{1}{2a} \ln \left| \frac{a+u}{a-u} \right| + C$$

# TRIGONOMETRY

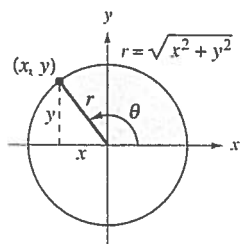
## Definition of the Six Trigonometric Functions

Right triangle definitions, where  $0 < \theta < \pi/2$ .

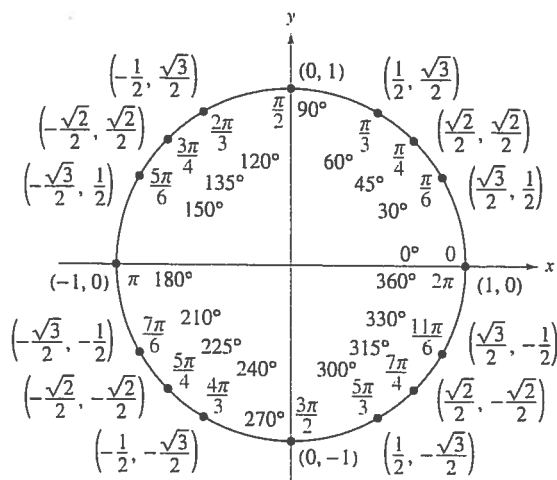


$$\begin{aligned} \sin \theta &= \frac{\text{opp}}{\text{hyp}} & \csc \theta &= \frac{\text{hyp}}{\text{opp}} \\ \cos \theta &= \frac{\text{adj}}{\text{hyp}} & \sec \theta &= \frac{\text{hyp}}{\text{adj}} \\ \tan \theta &= \frac{\text{opp}}{\text{adj}} & \cot \theta &= \frac{\text{adj}}{\text{opp}} \end{aligned}$$

Circular function definitions, where  $\theta$  is any angle.



$$\begin{aligned} \sin \theta &= \frac{y}{r} & \csc \theta &= \frac{r}{y} \\ \cos \theta &= \frac{x}{r} & \sec \theta &= \frac{r}{x} \\ \tan \theta &= \frac{y}{x} & \cot \theta &= \frac{x}{y} \end{aligned}$$



## Reciprocal Identities

$$\begin{aligned} \sin x &= \frac{1}{\csc x} & \sec x &= \frac{1}{\cos x} & \tan x &= \frac{1}{\cot x} \\ \csc x &= \frac{1}{\sin x} & \cos x &= \frac{1}{\sec x} & \cot x &= \frac{1}{\tan x} \end{aligned}$$

## Quotient Identities

$$\tan x = \frac{\sin x}{\cos x} \quad \cot x = \frac{\cos x}{\sin x}$$

## Pythagorean Identities

$$\begin{aligned} \sin^2 x + \cos^2 x &= 1 \\ 1 + \tan^2 x &= \sec^2 x & 1 + \cot^2 x &= \csc^2 x \end{aligned}$$

## Cofunction Identities

$$\begin{aligned} \sin\left(\frac{\pi}{2} - x\right) &= \cos x & \cos\left(\frac{\pi}{2} - x\right) &= \sin x \\ \csc\left(\frac{\pi}{2} - x\right) &= \sec x & \tan\left(\frac{\pi}{2} - x\right) &= \cot x \\ \sec\left(\frac{\pi}{2} - x\right) &= \csc x & \cot\left(\frac{\pi}{2} - x\right) &= \tan x \end{aligned}$$

## Even/Odd Identities

$$\begin{aligned} \sin(-x) &= -\sin x & \cos(-x) &= \cos x \\ \csc(-x) &= -\csc x & \tan(-x) &= -\tan x \\ \sec(-x) &= \sec x & \cot(-x) &= -\cot x \end{aligned}$$

## Sum and Difference Formulas

$$\begin{aligned} \sin(u \pm v) &= \sin u \cos v \pm \cos u \sin v \\ \cos(u \pm v) &= \cos u \cos v \mp \sin u \sin v \\ \tan(u \pm v) &= \frac{\tan u \pm \tan v}{1 \mp \tan u \tan v} \end{aligned}$$

## Double-Angle Formulas

$$\begin{aligned} \sin 2u &= 2 \sin u \cos u \\ \cos 2u &= \cos^2 u - \sin^2 u = 2 \cos^2 u - 1 = 1 - 2 \sin^2 u \\ \tan 2u &= \frac{2 \tan u}{1 - \tan^2 u} \end{aligned}$$

## Power-Reducing Formulas

$$\begin{aligned} \sin^2 u &= \frac{1 - \cos 2u}{2} \\ \cos^2 u &= \frac{1 + \cos 2u}{2} \\ \tan^2 u &= \frac{1 - \cos 2u}{1 + \cos 2u} \end{aligned}$$

## Sum-to-Product Formulas

$$\begin{aligned} \sin u + \sin v &= 2 \sin\left(\frac{u+v}{2}\right) \cos\left(\frac{u-v}{2}\right) \\ \sin u - \sin v &= 2 \cos\left(\frac{u+v}{2}\right) \sin\left(\frac{u-v}{2}\right) \\ \cos u + \cos v &= 2 \cos\left(\frac{u+v}{2}\right) \cos\left(\frac{u-v}{2}\right) \\ \cos u - \cos v &= -2 \sin\left(\frac{u+v}{2}\right) \sin\left(\frac{u-v}{2}\right) \end{aligned}$$

## Product-to-Sum Formulas

$$\begin{aligned} \sin u \sin v &= \frac{1}{2} [\cos(u-v) - \cos(u+v)] \\ \cos u \cos v &= \frac{1}{2} [\cos(u-v) + \cos(u+v)] \\ \sin u \cos v &= \frac{1}{2} [\sin(u+v) + \sin(u-v)] \\ \cos u \sin v &= \frac{1}{2} [\sin(u+v) - \sin(u-v)] \end{aligned}$$