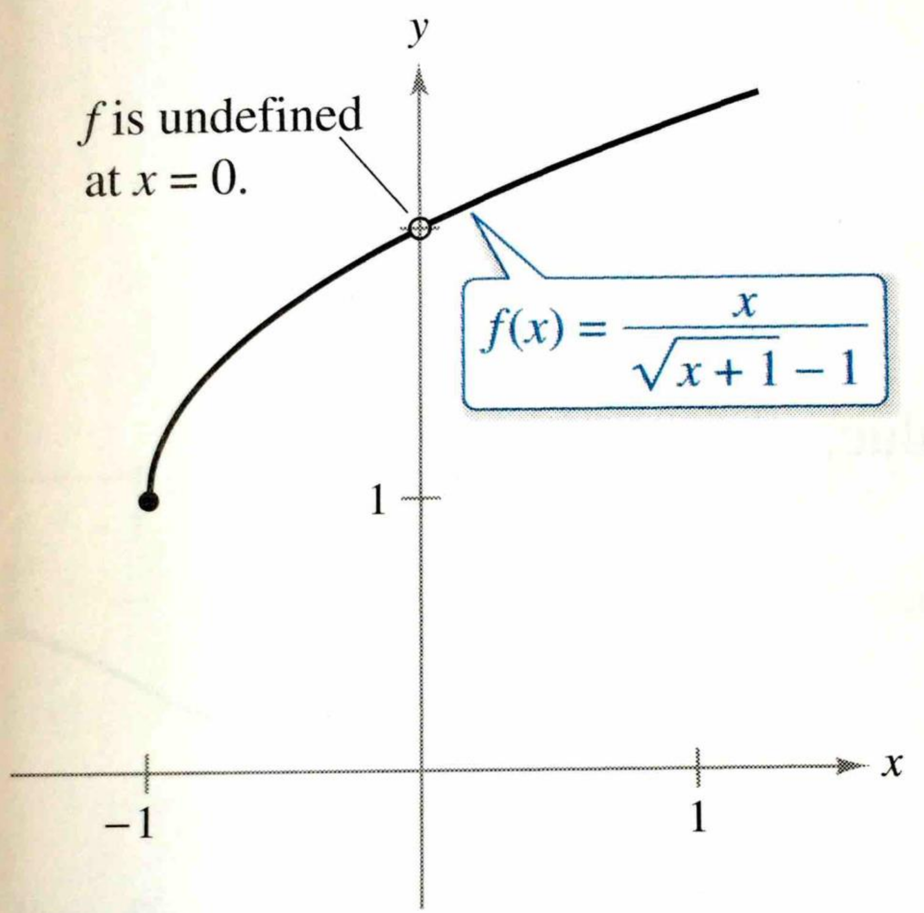
EXAMPLE 1 **Estimating a Limit Numerically**

Evaluate the function $f(x) = x/(\sqrt{x+1} - 1)$ at several x-values near 0 and use the results to estimate the limit

$$\lim_{x \to 0} \frac{x}{\sqrt{x+1} - 1}.$$



The limit of $f(x)$ as x approaches 0 is 2.

Figure 1.6

EXAMPLE 2 Finding a Limit

Find the limit of $f(x)$ as x approaches 2, where

$$f(x) = \begin{cases} 1, & x \neq 2 \\ 0, & x = 2 \end{cases}.$$

EXAMPLE 3 Different Right and Left Behavior

Show that the limit $\lim_{x \to 0} \frac{|x|}{x}$ does not exist.

EXAMPLE 4 Unbounded Behavior

Discuss the existence of the limit $\lim_{x \to 0} \frac{1}{x^2}$.

EXAMPLE 5 **Oscillating Behavior**

See LarsonCalculus.com for an interactive ve

Discuss the existence of the limit $\lim_{x \to 0} \sin \frac{1}{x}$.

x	0	0.001	0.01	0.1
$\frac{1}{x}$				
$\sin \frac{1}{x}$				

Step One: Choose some values of x getting closer to zero and place them in the top row (These are given above)

Step Two: Find the reciprocal of 0.001, 0.01 and 0.1 and write in row 2.

Step Three: Consider what is happening to the graph of our sine curve over each interval.

a) The period of the sine curve is __________
b) The range of the sine curve is ___________
c) How much closer did we move toward zero over the first interval (from 0.1 to 0.01) _____________?
d) During that same time, how many complete cycles did your sine curve make_________?
e) How much closer did we move toward zero over the second interval (from 0.01 to 0.001) _____________?
f) During that same time, how many complete cycles did your sine curve make_________?

Conclusion: What is happening to our sine curve as x approaches zero?____

____

Common Types of Behavior Associated with Nonexistence of a Limit

1. $f(x)$ approaches a different number from the right side of c than it approaches from the left side.
2. $f(x)$ increases or decreases without bound as x approaches c.
3. $f(x)$ oscillates between two fixed values as x approaches c.

Definition of Limit

Let f be a function defined on an open interval containing c (except possibly at c), and let L be a real number. The statement

$$\lim_{x \to c} f(x) = L$$

means that for each $\varepsilon > 0$ there exists a $\delta > 0$ such that if

$$0 < |x - c| < \delta$$

then

$$|f(x) - L| < \varepsilon.$$

EXAMPLE 7 Using the ε-δ Definition of Limit

Use the ε-δ definition of limit to prove that

$$\lim_{x \to 2} (3x - 2) = 4.$$

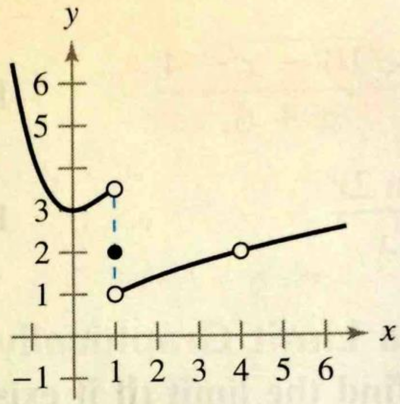
Graphical Reasoning **In Exercises 23 and 24, use the graph of the function *f* to decide whether the value of the given quantity exists. If it does, find it. If not, explain why.**

23. (a) $f(1)$

(b) $\lim_{x \to 1} f(x)$

(c) $f(4)$

(d) $\lim_{x \to 4} f(x)$



24. (a) $f(-2)$

(b) $\lim_{x \to -2} f(x)$

(c) $f(0)$

(d) $\lim_{x \to 0} f(x)$

(e) $f(2)$

(f) $\lim_{x \to 2} f(x)$

(g) $f(4)$

(h) $\lim_{x \to 4} f(x)$