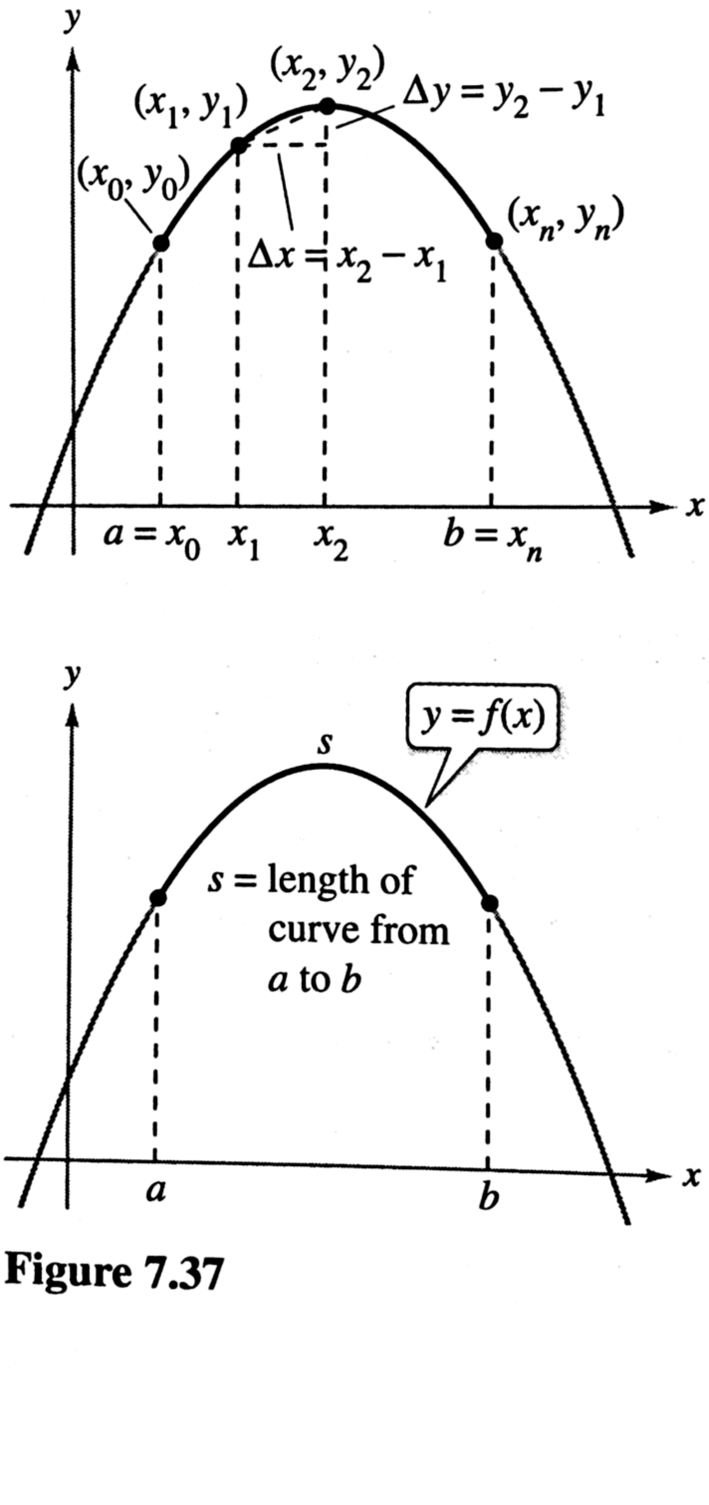

y
(x_2, y_2)
(x_1, y_1)
$\Delta y = y_2 - y_1$
(x_0, y_0)
(x_n, y_n)
$\Delta x = x_2 - x_1$
x
$a = x_0$
x_1
x_2
$b = x_n$
y
$y = f(x)$
s
s = length of curve from a to b
x
a
b

Figure 7.37

Definition of Arc Length

Let the function $y = f(x)$ represent a smooth curve on the interval $[a, b]$. The **arc length** of f between a and b is

$$s = \int_a^b \sqrt{1 + [f'(x)]^2}\, dx.$$

Similarly, for a smooth curve $x = g(y)$, the **arc length** of g between c and d is

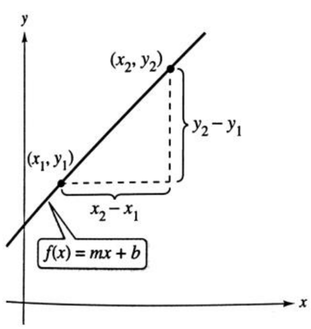
$$s = \int_c^d \sqrt{1 + [g'(y)]^2}\, dy.$$

EXAMPLE 1 The Length of a Line Segment

Find the arc length from (x_1, y_1) to (x_2, y_2) on the graph of

$$f(x) = mx + b$$

as shown in Figure 7.38.



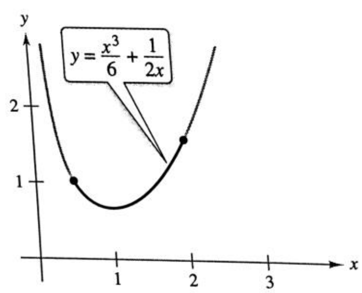
The formula for the arc length of the graph of f from (x_1, y_1) to (x_2, y_2) is the same as the standard Distance Formula.
Figure 7.38

EXAMPLE 2 Finding Arc Length

Find the arc length of the graph of

$$y = \frac{x^3}{6} + \frac{1}{2x}$$

on the interval $\left[\frac{1}{2}, 2\right]$, as shown in Figure 7.39.

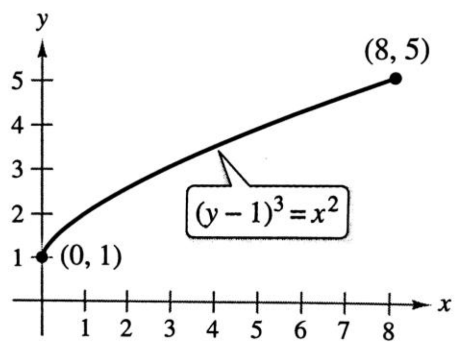


The arc length of the graph of y on $\left[\frac{1}{2}, 2\right]$

Figure 7.39

EXAMPLE 3 **Finding Arc Length**

Find the arc length of the graph of $(y-1)^3 = x^2$ on the interval $[0, 8]$, as shown in Figure 7.40.



The arc length of the graph of y on $[0, 8]$

Figure 7.40

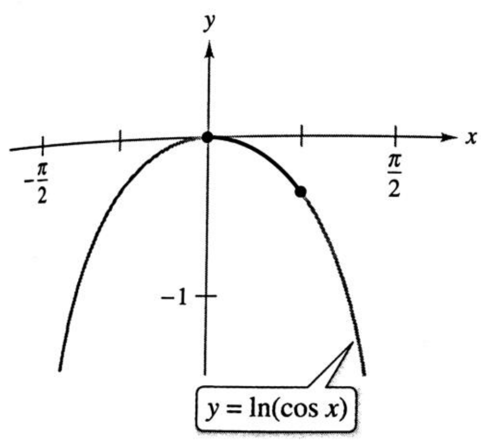
EXAMPLE 4 Finding Arc Length

See LarsonCalculus.com for an interactive version of this type of example.

Find the arc length of the graph of

$$y = \ln(\cos x)$$

from $x = 0$ to $x = \pi/4$, as shown in Figure 7.41.



The arc length of the graph of y on $\left[0, \frac{\pi}{4}\right]$

Figure 7.41

EXAMPLE 5 **Length of a Cable**

An electric cable is hung between two towers that are 200 feet apart, as shown in Figure 7.42. The cable takes the shape of a catenary whose equation is

$$y = 75(e^{x/150} + e^{-x/150}) = 150 \cosh \frac{x}{150}.$$

Find the arc length of the cable between the two towers.

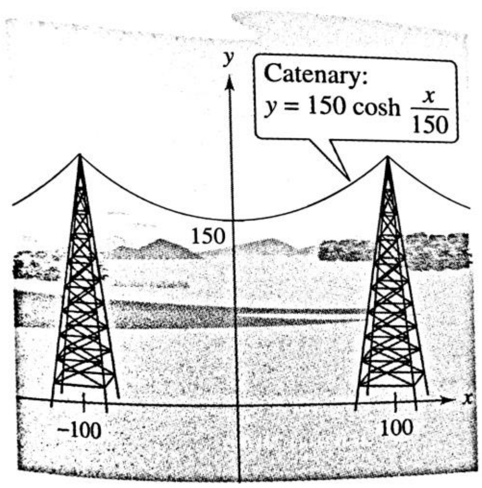
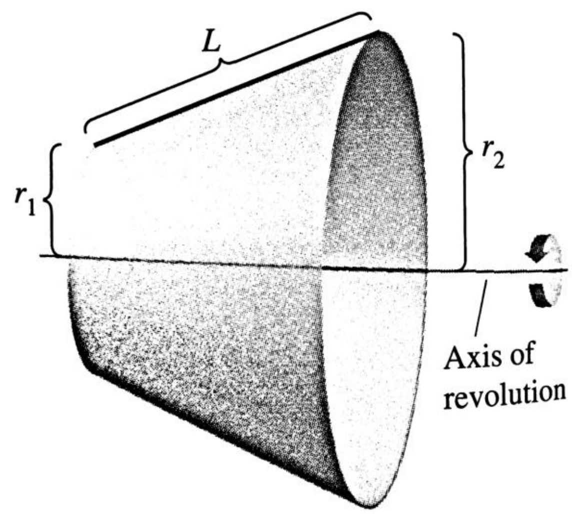


Figure 7.42

Definition of Surface of Revolution

When the graph of a continuous function is revolved about a line, the resulting surface is a **surface of revolution.**



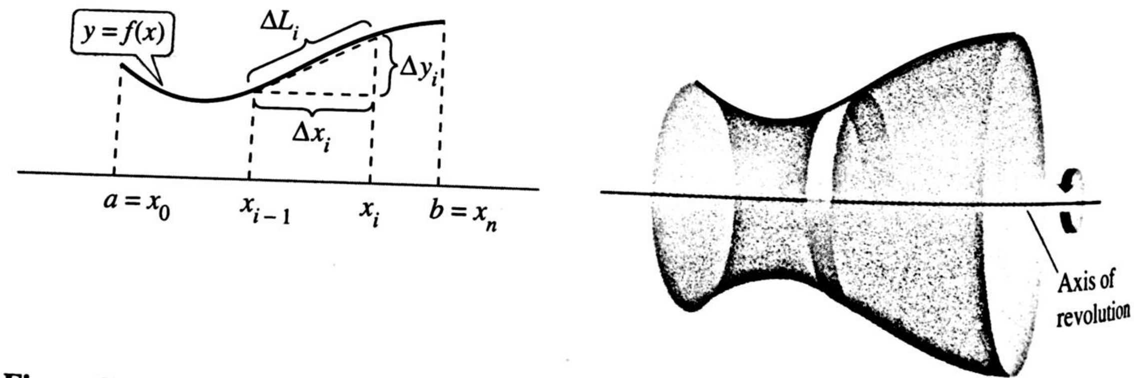


Figure 7.43

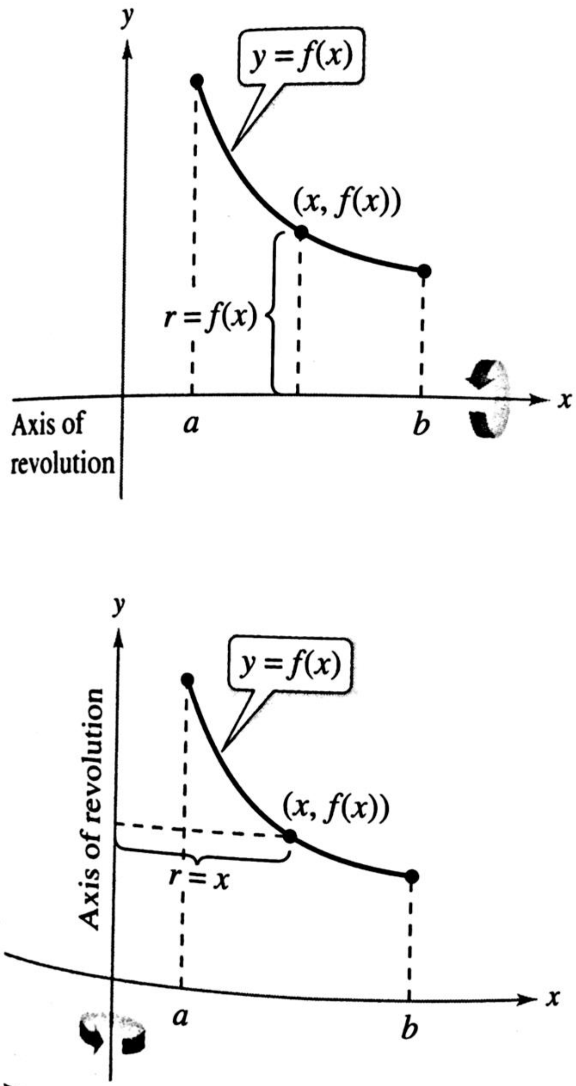

y
y = f(x)
(x, f(x))
r = f(x)
x
a
b
Axis of
revolution
y
y = f(x)
(x, f(x))
Axis of revolution
r = x
x
a
b

Figure 7.44

Definition of the Area of a Surface of Revolution

Let $y = f(x)$ have a continuous derivative on the interval $[a, b]$. The area S of the surface of revolution formed by revolving the graph of f about a horizontal or vertical axis is

$$S = 2\pi \int_a^b r(x)\sqrt{1 + [f'(x)]^2}\, dx \qquad y \text{ is a function of } x.$$

where $r(x)$ is the distance between the graph of f and the axis of revolution. If $x = g(y)$ on the interval $[c, d]$, then the surface area is

$$S = 2\pi \int_c^d r(y)\sqrt{1 + [g'(y)]^2}\, dy \qquad x \text{ is a function of } y.$$

where $r(y)$ is the distance between the graph of g and the axis of revolution.

EXAMPLE 6 The Area of a Surface of Revolution

Find the area of the surface formed by revolving the graph of $f(x) = x^3$ on the interval $[0, 1]$ about the x-axis, as shown in Figure 7.45.

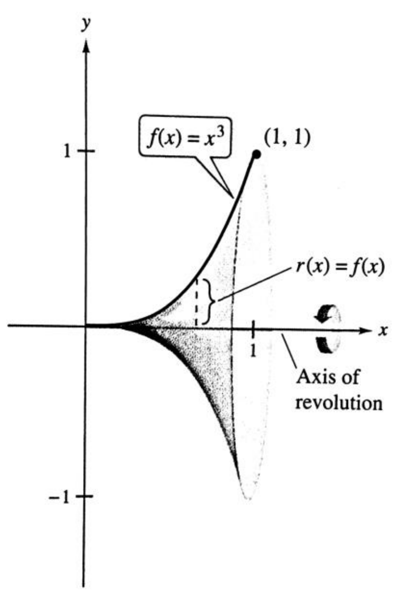


Figure 7.45

EXAMPLE 7 The Area of a Surface of Revolution

Find the area of the surface formed by revolving the graph of $f(x) = x^2$ on the interval $\left[0, \sqrt{2}\right]$ about the y-axis, as shown in the figure below.