

Definition of Definite Integral

If f is defined on the closed interval $[a, b]$ and the limit of Riemann sums over partitions Δ

$$\lim_{\|\Delta\| \rightarrow 0} \sum_{i=1}^n f(c_i) \Delta x_i$$

exists (as described above), then f is said to be **integrable** on $[a, b]$ and the limit is denoted by

$$\bullet \triangleright \quad \lim_{\|\Delta\| \rightarrow 0} \sum_{i=1}^n f(c_i) \Delta x_i = \int_a^b f(x) dx.$$

The limit is called the **definite integral** of f from a to b . The number a is the **lower limit** of integration, and the number b is the **upper limit** of integration.

THEOREM 4.5 The Definite Integral as the Area of a Region

If f is continuous and nonnegative on the closed interval $[a, b]$, then the area of the region bounded by the graph of f , the x -axis, and the vertical lines $x = a$ and $x = b$ is

$$\text{Area} = \int_a^b f(x) dx.$$

(See Figure 4.22.)

EXAMPLE 3 Areas of Common Geometric Figures

Sketch the region corresponding to each definite integral. Then evaluate each integral using a geometric formula.

a. $\int_1^3 4 \, dx$ b. $\int_0^3 (x + 2) \, dx$ c. $\int_{-2}^2 \sqrt{4 - x^2} \, dx$

Definitions of Two Special Definite Integrals

1. If f is defined at $x = a$, then $\int_a^a f(x) \, dx = 0$.
2. If f is integrable on $[a, b]$, then $\int_b^a f(x) \, dx = -\int_a^b f(x) \, dx$.

EXAMPLE 4 Evaluating Definite Integrals

•••▶ See *LarsonCalculus.com* for an interactive version of this type of example.

Evaluate each definite integral.

a. $\int_{\pi}^{\pi} \sin x \, dx$ b. $\int_3^0 (x + 2) \, dx$

THEOREM 4.6 Additive Interval Property

If f is integrable on the three closed intervals determined by a , b , and c , then

$$\int_a^b f(x) \, dx = \int_a^c f(x) \, dx + \int_c^b f(x) \, dx.$$

EXAMPLE 5

$$\int_{-1}^1 |x| \, dx =$$

THEOREM 4.7 Properties of Definite Integrals

If f and g are integrable on $[a, b]$ and k is a constant, then the functions kf and $f \pm g$ are integrable on $[a, b]$, and

$$1. \int_a^b kf(x) dx = k \int_a^b f(x) dx$$

$$2. \int_a^b [f(x) \pm g(x)] dx = \int_a^b f(x) dx \pm \int_a^b g(x) dx.$$

EXAMPLE 6**Evaluation of a Definite Integral**

Evaluate $\int_1^3 (-x^2 + 4x - 3) dx$ using each of the following values.

$$\int_1^3 x^2 dx = \frac{26}{3}, \quad \int_1^3 x dx = 4, \quad \int_1^3 dx = 2$$

THEOREM 4.8 Preservation of Inequality

1. If f is integrable and nonnegative on the closed interval $[a, b]$, then

$$0 \leq \int_a^b f(x) dx.$$

2. If f and g are integrable on the closed interval $[a, b]$ and $f(x) \leq g(x)$ for every x in $[a, b]$, then

$$\int_a^b f(x) dx \leq \int_a^b g(x) dx.$$

A proof of this theorem is given in Appendix A.

See LarsonCalculus.com for Bruce Edwards's video of this proof.