Trigonometric Functions at $\frac{\pi}{4}$

$$\sin\frac{\pi}{4} = \frac{\sqrt{2}}{2} \qquad \csc\frac{\pi}{4} = \sqrt{2}$$

$$\cos\frac{\pi}{4} = \frac{\sqrt{2}}{2} \qquad \sec\frac{\pi}{4} = \sqrt{2}$$

$$\tan\frac{\pi}{4} = 1 \qquad \cot\frac{\pi}{4} = 1$$

The Domain and Range of the Sine and Cosine Functions

The domain of the sine function and the cosine function is $(-\infty, \infty)$, the set of all real numbers. The range of these functions is $[-1, 1]$, the set of all real numbers from -1 to 1, inclusive.

Definitions of the Trigonometric Functions in Terms of a Unit Circle

If t is a real number and $P = (x, y)$ is a point on the unit circle that corresponds to t, then

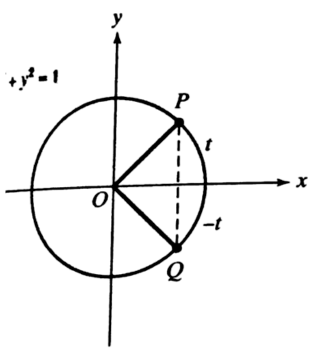
$$\sin t = y \qquad \csc t = \frac{1}{y}, y \neq 0$$

$$\cos t = x \qquad \sec t = \frac{1}{x}, x \neq 0$$

$$\tan t = \frac{y}{x}, x \neq 0 \qquad \cot t = \frac{x}{y}, y \neq 0.$$

Name	Abbreviation	Name	Abbreviation
sine	sin	cosecant	csc
cosine	cos	secant	sec
tangent	tan	cotangent	cot

Use even and odd trigonometric functions.



IRE 4.27

Even and Odd Trigonometric Functions

We have seen that a function is even if $f(-t) = f(t)$ and odd if $f(-t) = -f(t)$. We can use **Figure 4.27** to show that the cosine function is an even function and the sine function is an odd function. By definition, the coordinates of the points P and Q in **Figure 4.27** are as follows:

$$P: (\cos t, \sin t)$$
$$Q: (\cos(-t), \sin(-t)).$$

In **Figure 4.27**, the x-coordinates of P and Q are the same. Thus,

$$\cos(-t) = \cos t.$$

This shows that the cosine function is an even function. By contrast, the y-coordinates of P and Q are negatives of each other. Thus,

$$\sin(-t) = -\sin t.$$

This shows that the sine function is an odd function.

Even and Odd Trigonometric Functions

The cosine and secant functions are *even*.

$$\cos(-t) = \cos t \qquad \sec(-t) = \sec t$$

The sine, cosecant, tangent, and cotangent functions are *odd*.

$$\sin(-t) = -\sin t \qquad \csc(-t) = -\csc t$$
$$\tan(-t) = -\tan t \qquad \cot(-t) = -\cot t$$

EXAMPLE 4 Using Even and Odd Functions to Find Values of Trigonometric Functions

Find the value of each trigonometric function:

a. $\cos\left(-\frac{\pi}{4}\right)$ **b.** $\tan\left(-\frac{\pi}{4}\right)$.

Reciprocal Identities

$$\sin t = \frac{1}{\csc t} \qquad \csc t = \frac{1}{\sin t}$$

$$\cos t = \frac{1}{\sec t} \qquad \sec t = \frac{1}{\cos t}$$

$$\tan t = \frac{1}{\cot t} \qquad \cot t = \frac{1}{\tan t}$$

Quotient Identities

$$\tan t = \frac{\sin t}{\cos t} \qquad \cot t = \frac{\cos t}{\sin t}$$

... ythagorean identities.

Pythagorean Identities

$$\sin^2 t + \cos^2 t = 1 \qquad 1 + \tan^2 t = \sec^2 t \qquad 1 + \cot^2 t = \csc^2 t$$

Definition of a Periodic Function

A function f is **periodic** if there exists a positive number p such that

$$f(t + p) = f(t)$$

for all t in the domain of f. The smallest positive number p for which f is periodic is called the **period** of f.

Periodic Properties of the Sine and Cosine Functions

$$\sin(t + 2\pi) = \sin t \quad \text{and} \quad \cos(t + 2\pi) = \cos t$$

The sine and cosine functions are periodic functions and have period 2π.

Periodic Properties of the Tangent and Cotangent Functions

$$\tan(t + \pi) = \tan t \quad \text{and} \quad \cot(t + \pi) = \cot t$$

The tangent and cotangent functions are periodic functions and have period π.

Pythagorean Identities

$$\sin^2 t + \cos^2 t = 1 \qquad 1 + \tan^2 t = \sec^2 t \qquad 1 + \cot^2 t = \csc^2 t$$

Check Point 6 Given that $\sin t = \frac{1}{2}$ and $0 \le t < \frac{\pi}{2}$, find the value of $\cos t$ using a trigonometric identity. $\frac{\sqrt{3}}{2}$

Application Exercises

81. The number of hours of daylight, H, on day t of any given year (on January 1, $t = 1$) in Fairbanks, Alaska, can be modeled by the function

$$H(t) = 12 + 8.3 \sin\left[\frac{2\pi}{365}(t - 80)\right].$$

a. March 21, the 80th day of the year, is the spring equinox. Find the number of hours of daylight in Fairbanks on this day. 12 hr

b. June 21, the 172nd day of the year, is the summer solstice, the day with the maximum number of hours of daylight. To the nearest tenth of an hour, find the number of hours of daylight in Fairbanks on this day. 20.3 hr

c. December 21, the 355th day of the year, is the winter solstice, the day with the minimum number of hours of daylight. Find, to the nearest tenth of an hour, the number of hours of daylight in Fairbanks on this day. 3.7 hr