To prove the Law of Cosines, we place triangle $ABC$ in a rectangular coordinate system. **Figure 6.14** shows a triangle with three acute angles. The vertex $A$ is at the origin and side $c$ lies along the positive $x$-axis. The coordinates of $C$ are $(x, y)$. Using the right triangle that contains angle $A$, we apply the definitions of the cosine and the sine.

$$\cos A = \frac{x}{b} \qquad \sin A = \frac{y}{b}$$

$$x = b\cos A \qquad y = b\sin A$$ Multiply both sides of each equation by $b$ and solve for $x$ and $y$, respectively.

Thus, the coordinates of $C$ are $(x, y) = (b\cos A, b\sin A)$. Although triangle $ABC$ in **Figure 6.14** shows angle $A$ as an acute angle, if $A$ were obtuse, the coordinates of $C$ would still be $(b\cos A, b\sin A)$. This means that our proof applies to both kinds of oblique triangles.

We now apply the distance formula to the side of the triangle with length $a$. Notice that $a$ is the distance from $(x, y)$ to $(c, 0)$.

| | |
|---|---|
| $a = \sqrt{(x - c)^2 + (y - 0)^2}$ | Use the distance formula. |
| $a^2 = (x - c)^2 + y^2$ | Square both sides of the equation. |
| $a^2 = (b\cos A - c)^2 + (b\sin A)^2$ | $x = b\cos A$ and $y = b\sin A$. |
| $a^2 = b^2\cos^2 A - 2bc\cos A + c^2 + b^2\sin^2 A$ | Square the two expressions. |
| $a^2 = b^2\sin^2 A + b^2\cos^2 A + c^2 - 2bc\cos A$ | Rearrange terms. |
| $a^2 = b^2(\sin^2 A + \cos^2 A) + c^2 - 2bc\cos A$ | Factor $b^2$ from the first two terms. |
| $a^2 = b^2 + c^2 - 2bc\cos A$ | $\sin^2 A + \cos^2 A = 1$ |

The resulting equation is one of the three formulas for the Law of Cosines. The other two formulas are derived in a similar manner.

**Check Point 1** Solve the triangle shown in **Figure 6.16** with $A = 120°$, $b = 7$, and $c = 8$. Round as in Example 1. $a = 13, B \approx 28°, c \approx 32°$

**Check Point 2** Solve triangle $ABC$ if $a = 8$, $b = 10$, and $c = 5$. Round angle measures to the nearest degree. $A \approx 52°, B \approx 98°, C \approx 30°$

## Applications of the Law of Cosines

Applied problems involving SAS and SSS triangles can be solved using the Law of Cosines.

**EXAMPLE 3** An Application of the Law of Cosines

Two airplanes leave an airport at the same time on different runways. One flies on a bearing of N66°W at 325 miles per hour. The other airplane flies on a bearing of S26°W at 300 miles per hour. How far apart will the airplanes be after two hours?