**THEOREM 9.12 Direct Comparison Test**

Let $0 < a_n \leq b_n$ for all $n$.

**1.** If $\sum_{n=1}^{\infty} b_n$ converges, then $\sum_{n=1}^{\infty} a_n$ converges.

**2.** If $\sum_{n=1}^{\infty} a_n$ diverges, then $\sum_{n=1}^{\infty} b_n$ diverges.

## EXAMPLE 1 Using the Direct Comparison Test

Determine the convergence or divergence of

$$\sum_{n=1}^{\infty} \frac{1}{2 + 3^n}.$$

## EXAMPLE 2 Using the Direct Comparison Test

See LarsonCalculus.com for an interactive version of this type o

Determine the convergence or divergence of

$$\sum_{n=1}^{\infty} \frac{1}{2 + \sqrt{n}}.$$

**THEOREM 9.13 Limit Comparison Test**

If $a_n > 0$, $b_n > 0$, and

$$\lim_{n \to \infty} \frac{a_n}{b_n} = L$$

where $L$ is *finite and positive*, then

$$\sum_{n=1}^{\infty} a_n \quad \text{and} \quad \sum_{n=1}^{\infty} b_n$$

either both converge or both diverge.

## EXAMPLE 3 Using the Limit Comparison Test

Show that the general harmonic series below diverges.

$$\sum_{n=1}^{\infty} \frac{1}{an+b}, \quad a > 0, \quad b > 0$$

| Given Series | Comparison Series | Conclusion |
| --- | --- | --- |
| $\sum_{n=1}^{\infty} \frac{1}{3n^2 - 4n + 5}$ | $\sum_{n=1}^{\infty} \frac{1}{n^2}$ | Both series converge. |
| $\sum_{n=1}^{\infty} \frac{1}{\sqrt{3n-2}}$ | $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ | Both series diverge. |
| $\sum_{n=1}^{\infty} \frac{n^2 - 10}{4n^5 + n^3}$ | $\sum_{n=1}^{\infty} \frac{n^2}{n^5} = \sum_{n=1}^{\infty} \frac{1}{n^3}$ | Both series converge. |

## EXAMPLE 4 Using the Limit Comparison Test

Determine the convergence or divergence of

$$\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n^2 + 1}.$$

## EXAMPLE 5 Using the Limit Comparison Test

Determine the convergence or divergence of

$$\sum_{n=1}^{\infty} \frac{n2^n}{4n^3 + 1}.$$