**Definition of a Plane Curve**

If $f$ and $g$ are continuous functions of $t$ on an interval $I$, then the equations

$$x = f(t) \quad \text{and} \quad y = g(t)$$

are **parametric equations** and $t$ is the **parameter.** The set of points $(x, y)$ obtained as $t$ varies over the interval $I$ is the **graph** of the parametric equations. Taken together, the parametric equations and the graph are a **plane curve,** denoted by $C$.

**EXAMPLE 1** **Sketching a Curve**

Sketch the curve described by the parametric equations

$$x = f(t) = t^2 - 4$$

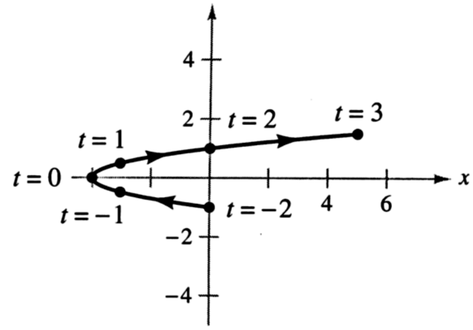
and

$$y = g(t) = \frac{t}{2}$$

where $-2 \le t \le 3$.

e parametric equations yield the points $(x, y)$
iown in the table.

| $t$ | $-2$ | $-1$ | $0$ | $1$ | $2$ | $3$ |
|---|---|---|---|---|---|---|
| $x$ | $0$ | $-3$ | $-4$ | $-3$ | $0$ | $5$ |
| $y$ | $-1$ | $-\frac{1}{2}$ | $0$ | $\frac{1}{2}$ | $1$ | $\frac{3}{2}$ |




Parametric equations:
$x = t^2 - 4$ and $y = \frac{t}{2}$, $-2 \le t \le 3$

y plotting these points in order of increasing

Parametric equations → Solve for $t$ in one equation. → Substitute into second equation. → Rectangular equation

$x = t^2 - 4$ | $t = 2y$ | $x = (2y)^2 - 4$ | $x = 4y^2 - 4$

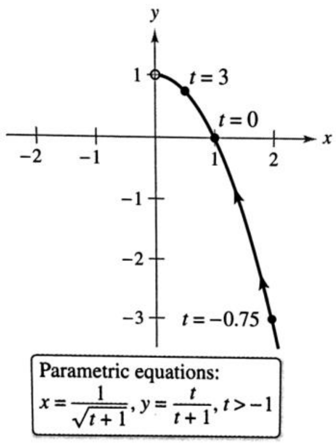
$y = t/2$

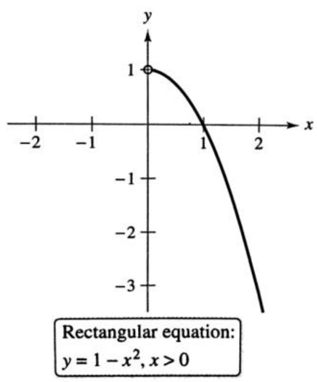
## EXAMPLE 2 Adjusting the Domain

Sketch the curve represented by the equations

$$x = \frac{1}{\sqrt{t+1}} \quad \text{and} \quad y = \frac{t}{t+1}, \quad t > -1$$

by eliminating the parameter and adjusting the domain of the re
equation.


y
x
1
-1
-2
-3
-2
-1
1
2
t = 3
t = 0
t = −0.75
Parametric equations:
$x = \frac{1}{\sqrt{t+1}}, y = \frac{t}{t+1}, t > -1$


y
x
1
-1
-2
-3
-2
-1
1
2
Rectangular equation:
$y = 1 - x^2, x > 0$

## EXAMPLE 3 Using Trigonometry to Eliminate a Parameter

*See LarsonCalculus.com for an interactive version of this type of example.*

Sketch the curve represented by

$$x = 3\cos\theta \quad \text{and} \quad y = 4\sin\theta, \quad 0 \le \theta \le 2\pi$$

by eliminating the parameter and finding the corresponding rectangular equation.

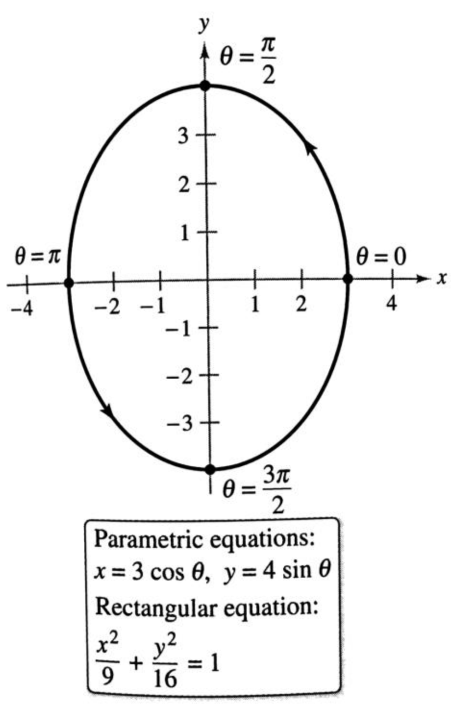



**Figure 10.23**

**EXAMPLE 4** **Finding Parametric Equations for a Given Graph**

Find a set of parametric equations that represents the graph of $y = 1 - x^2$, using each of the following parameters.

**a.** $t = x$ **b.** The slope $m = \frac{dy}{dx}$ at the point $(x, y)$

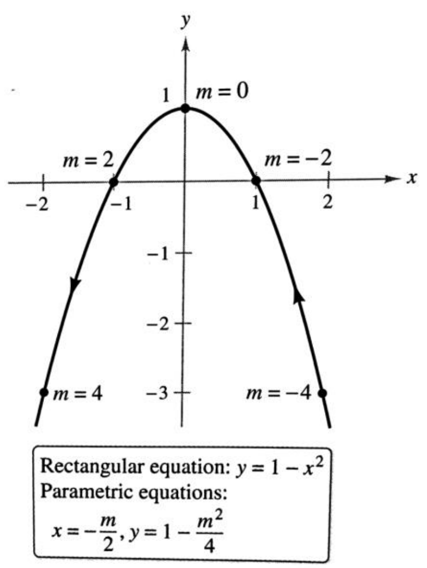



**Figure 10.24**

**EXAMPLE 5** **Parametric Equations for a Cycloid**

Determine the curve traced by a point $P$ on the circumference of a circle of radius $a$ rolling along a straight line in a plane. Such a curve is called a **cycloid.**

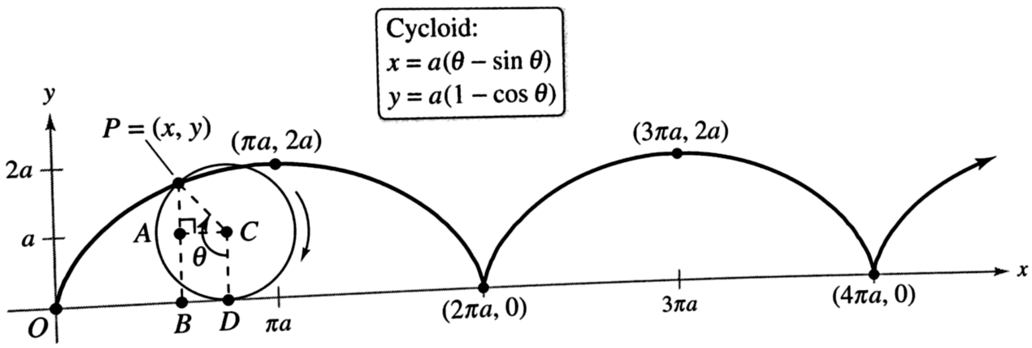


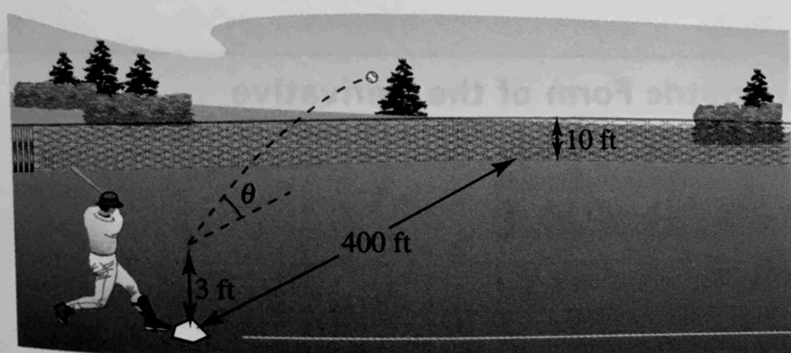

**Figure 10.25**

**Definition of a Smooth Curve**

A curve $C$ represented by $x = f(t)$ and $y = g(t)$ on an interval $I$ is called **smooth** when $f'$ and $g'$ are continuous on $I$ and not simultaneously 0, except possibly at the endpoints of $I$. The curve $C$ is called **piecewise smooth** when it is smooth on each subinterval of some partition of $I$.

**Projectile Motion** **In Exercises 79 and 80, consider a projectile launched at a height $h$ feet above the ground and at an angle $\theta$ with the horizontal. When the initial velocity is $v_0$ feet per second, the path of the projectile is modeled by the parametric equations $x = (v_0 \cos \theta)t$ and $y = h + (v_0 \sin \theta)t - 16t^2$.**

**79.** The center field fence in a ballpark is 10 feet high and 400 feet from home plate. The ball is hit 3 feet above the ground. It leaves the bat at an angle of $\theta$ degrees with the horizontal at a speed of 100 miles per hour (see figure).




(a) Write a set of parametric equations for the path of the ball.

(b) Use a graphing utility to graph the path of the ball when $\theta = 15°$. Is the hit a home run?

(c) Use a graphing utility to graph the path of the ball when $\theta = 23°$. Is the hit a home run?

(d) Find the minimum angle at which the ball must leave the bat in order for the hit to be a home run.

**80.** A rectan