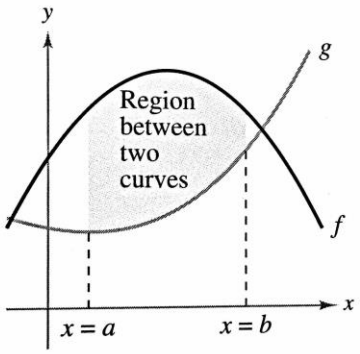
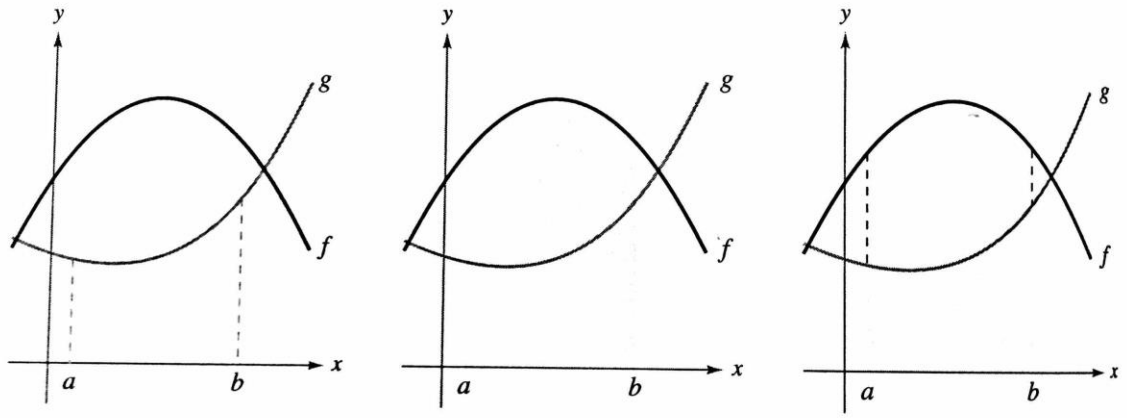


Figure 7.1



Area of region between f and g = Area of region under f − Area of region under g

$$\int_a^b [f(x) - g(x)]\,dx = \int_a^b f(x)\,dx - \int_a^b g(x)\,dx$$

Figure 7.2

Area of a Region Between Two Curves

If f and g are continuous on $[a, b]$ and $g(x) \leq f(x)$ for all x in $[a, b]$, then the area of the region bounded by the graphs of f and g and the vertical lines $x = a$ and $x = b$ is

$$A = \int_a^b [f(x) - g(x)]\,dx.$$

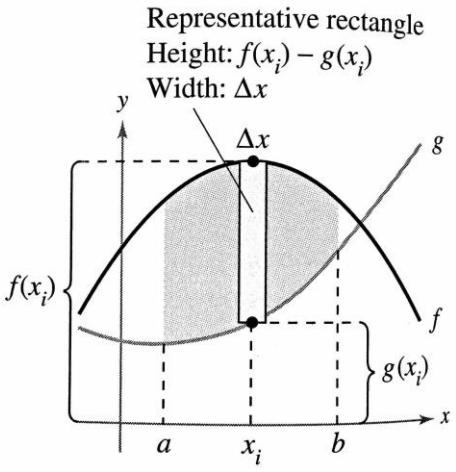
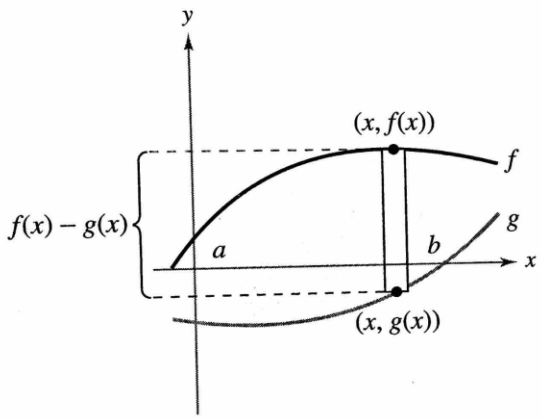


Figure 7.3



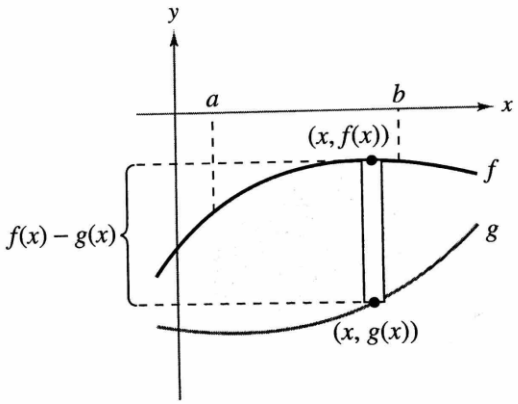
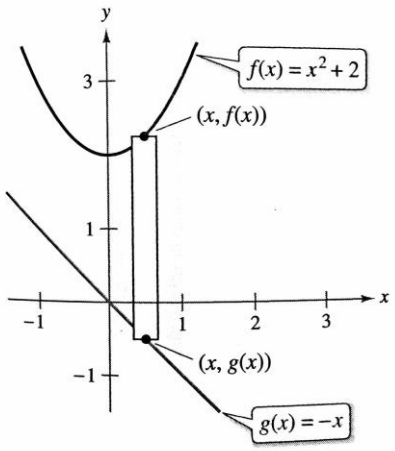


Figure 7.4

EXAMPLE 2 Finding the Area of a Region Between Two Curves

Find the area of the region bounded by the graphs of $y = x^2 + 2$, $y = -x$, $x = 0$, and $x = 1$.

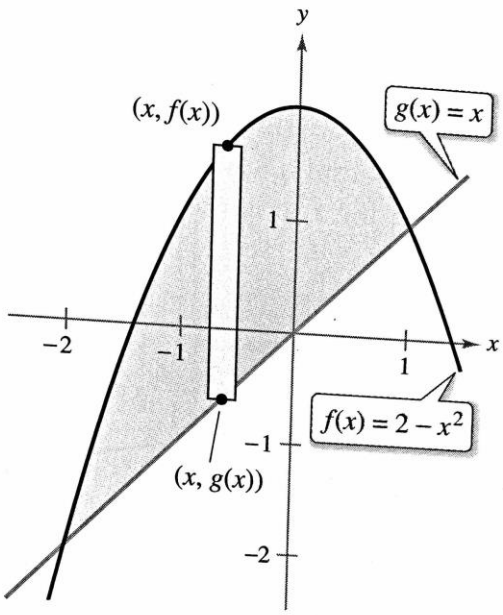


Region bounded by the graph of f, the graph of g, $x = 0$, and $x = 1$

Figure 7.5

EXAMPLE 2 **A Region Lying Between Two Intersecting Graphs**

Find the area of the region bounded by the graphs of $f(x) = 2 - x^2$ and $g(x) = x$.

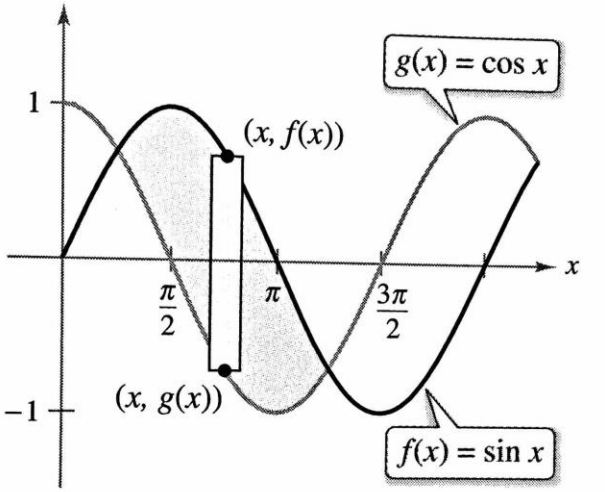


Region bounded by the graph of f and the graph of g

Figure 7.6

EXAMPLE 3 **A Region Lying Between Two Intersecting Graphs**

The sine and cosine curves intersect infinitely many times, bounding regions of equal areas, as shown in Figure 7.7. Find the area of one of these regions.



One of the regions bounded by the graphs of the sine and cosine functions

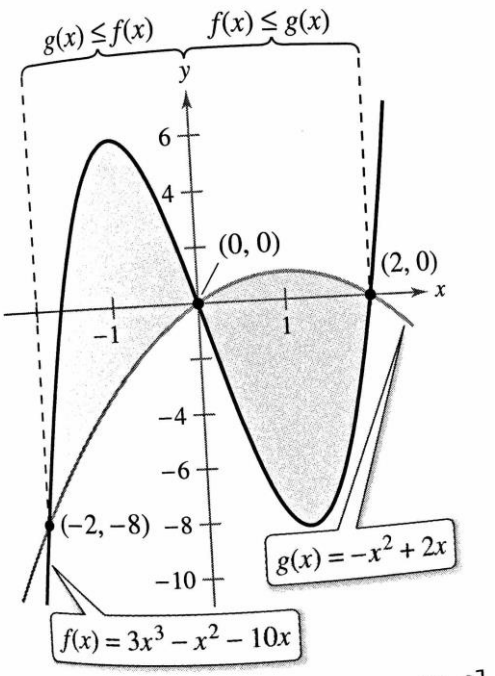
Figure 7.7

EXAMPLE 5 Curves That Intersect at More than Two Points

See LarsonCalculus.com for an interactive version of this type of example.

Find the area of the region between the graphs of

$$f(x) = 3x^3 - x^2 - 10x \quad \text{and} \quad g(x) = -x^2 + 2x.$$

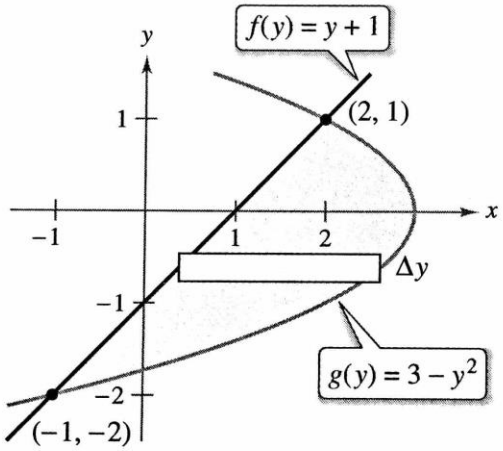


On $[-2, 0]$, $g(x) \leq f(x)$, and on $[0, 2]$, $f(x) \leq g(x)$.

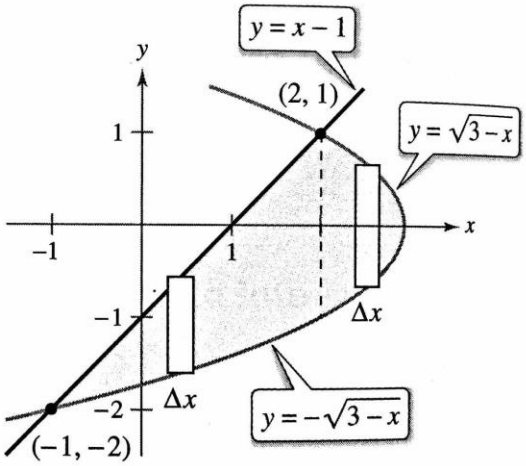
Figure 7.8

EXAMPLE 5 Horizontal Representative Rectangles

Find the area of the region bounded by the graphs of $x = 3 - y^2$ and $x = y + 1$.



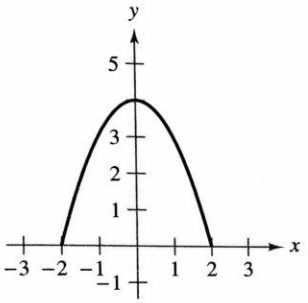
Horizontal rectangles (integration with respect to y)
Figure 7.9



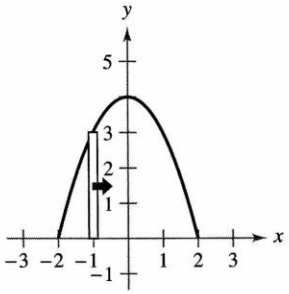
Vertical rectangles (integration with respect to x)
Figure 7.10

Integration as an Accumulation Process

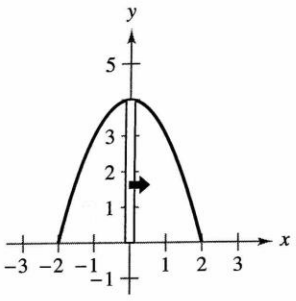
Find the area of the region bounded by the graph of $y = 4 - x^2$ and the x-axis. Describe the integration as an accumulation process.



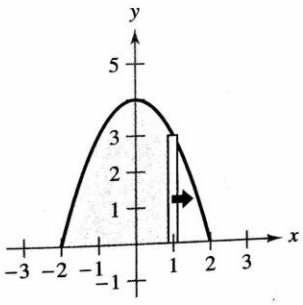
$$A = \int_{-2}^{-2} (4 - x^2)\,dx = 0$$



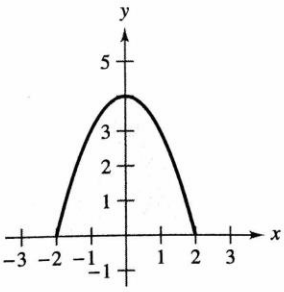
$$A = \int_{-2}^{-1} (4 - x^2)\,dx = \frac{5}{3}$$



$$A = \int_{-2}^{0} (4 - x^2)\,dx = \frac{16}{3}$$



$$A = \int_{-2}^{1} (4 - x^2)\,dx = 9$$



$$A = \int_{-2}^{2} (4 - x^2)\,dx = \frac{32}{3}$$

■