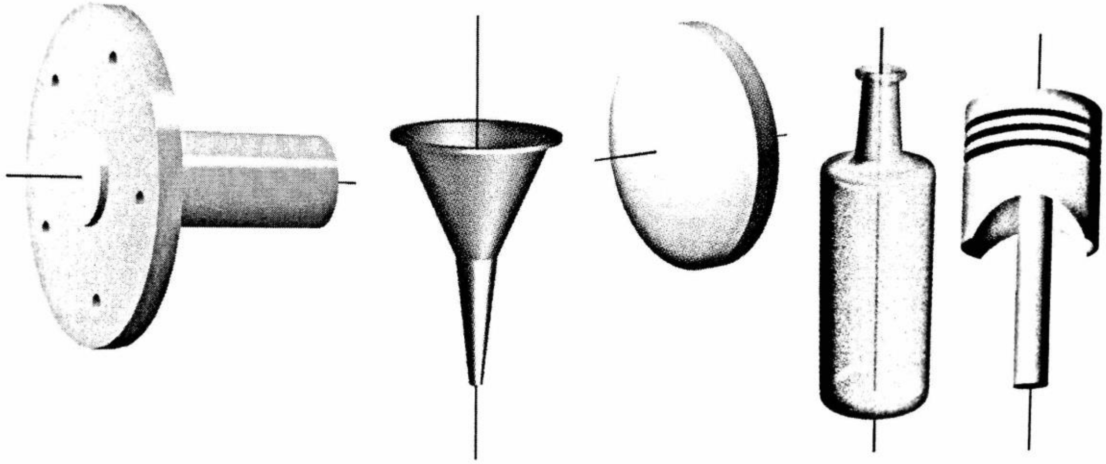
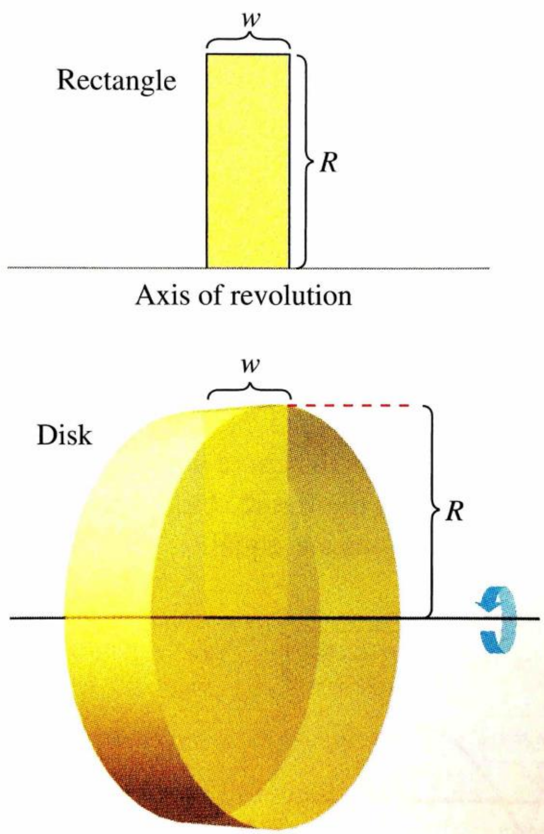


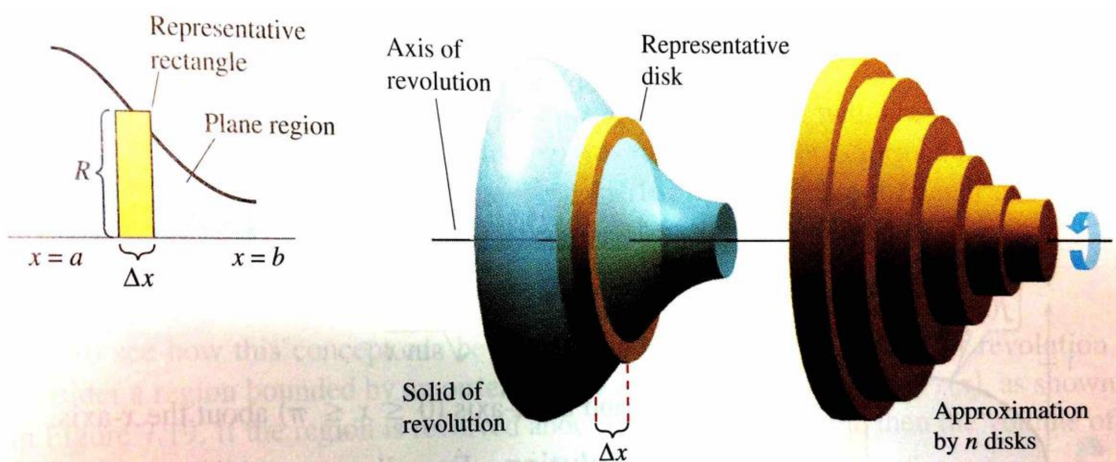
Solids of revolution
Figure 7.12

$$\Delta V = \pi R^2 \Delta x.$$



Volume of a disk: $\pi R^2 w$

Figure 7.13



Disk method

Figure 7.14

THE DISK METHOD

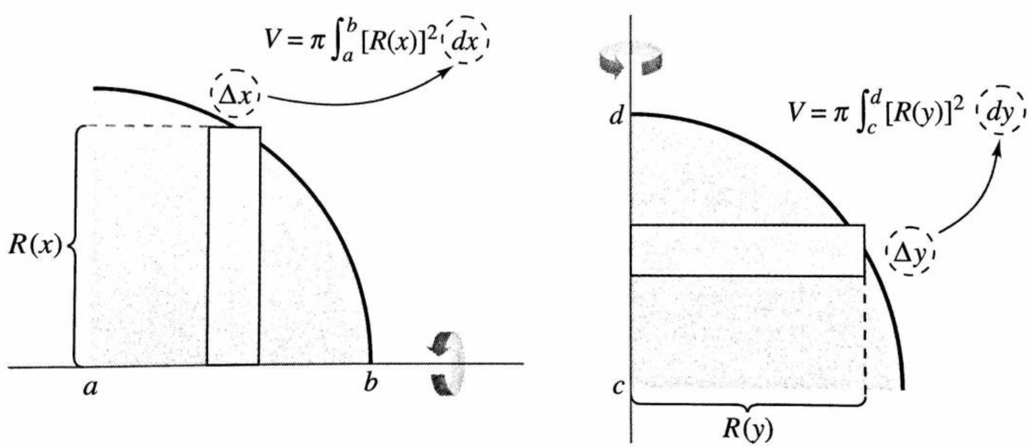
To find the volume of a solid of revolution with the **disk method,** use one of the formulas below. (See Figure 7.15.)

Horizontal Axis of Revolution

$$\text{Volume} = V = \pi\int_a^b [R(x)]^2\,dx$$

Vertical Axis of Revolution

$$\text{Volume} = V = \pi\int_c^d [R(y)]^2\,dy$$



Horizontal axis of revolution

Vertical axis of revolution

Figure 7.15

EXAMPLE 1 Using the Disk Method

Find the volume of the solid formed by revolving the region bounded by the graph of

$$f(x) = \sqrt{\sin x}$$

and the x-axis $(0 \le x \le \pi)$ about the x-axis.

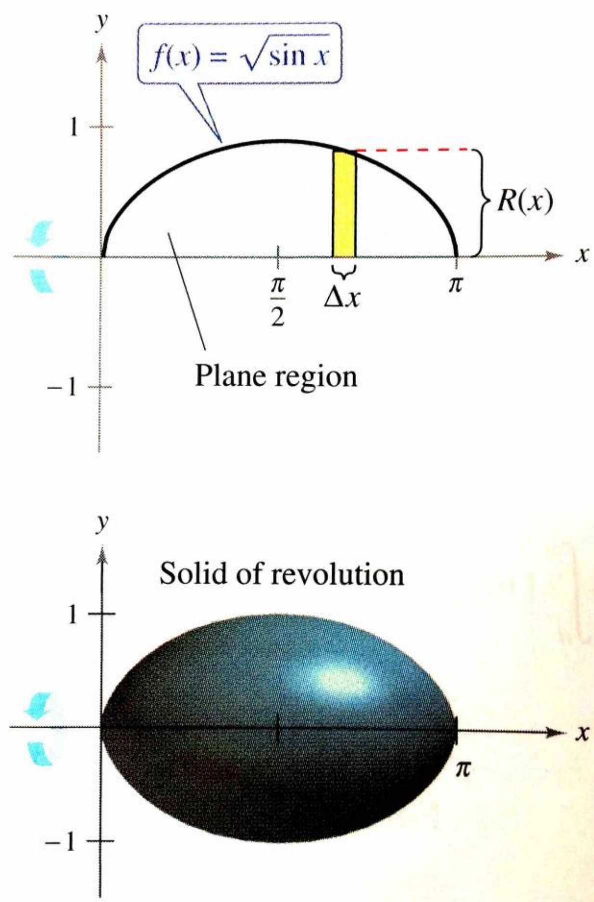


Figure 7.16

EXAMPLE 2 Using a Line That Is Not a Coordinate Axis

Find the volume of the solid formed by revolving the region bounded by the graphs of

$$f(x) = 2 - x^2$$

and $g(x) = 1$ about the line $y = 1$, as shown in Figure 7.17.

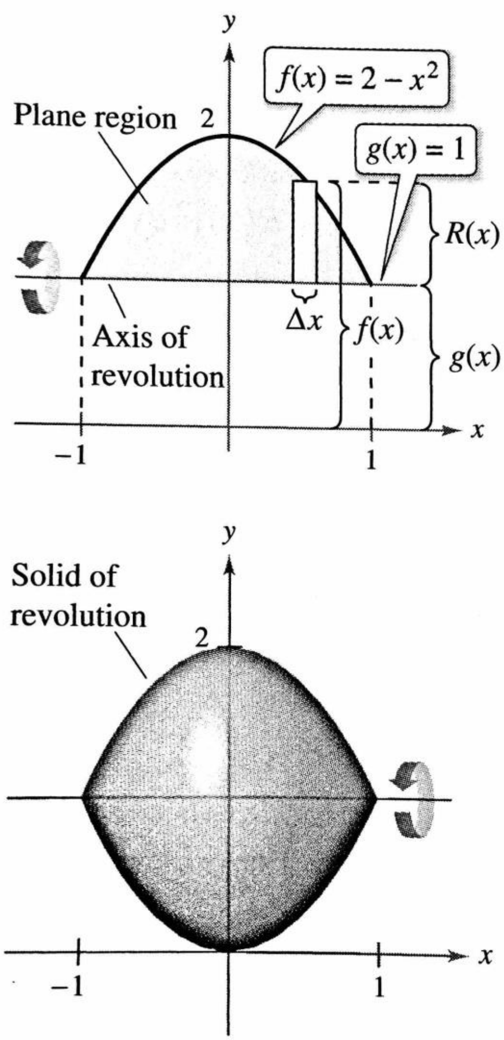


Figure 7.17

$$V = \pi \int_a^b \left([R(x)]^2 - [r(x)]^2\right) dx.$$

Washer method

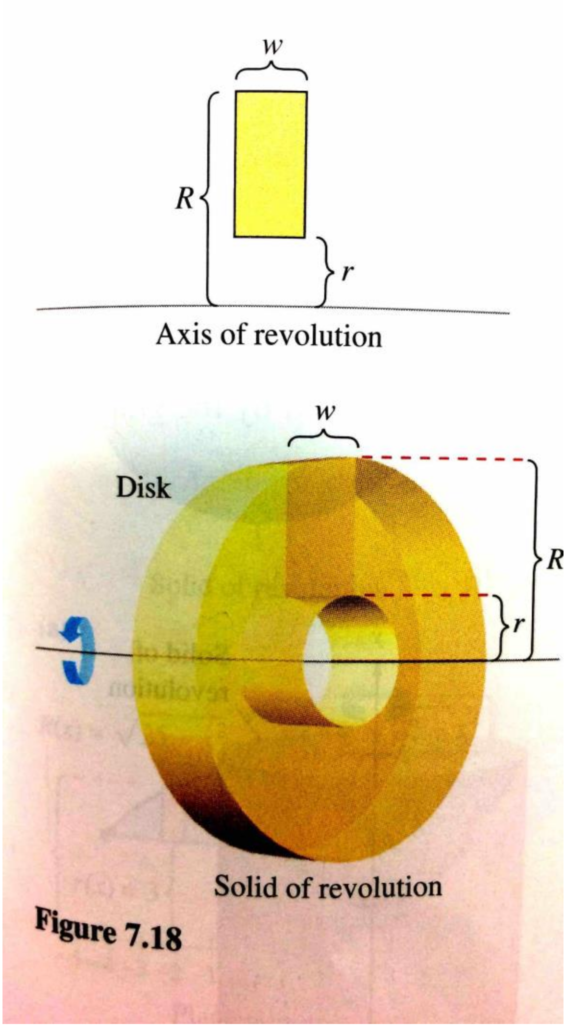


Figure 7.18

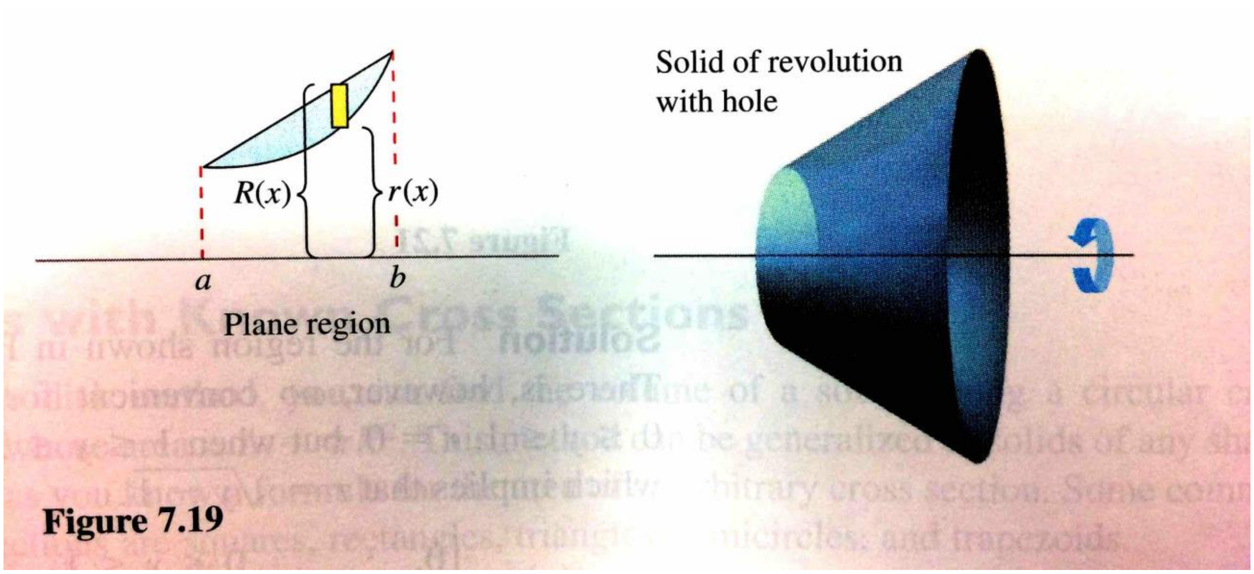


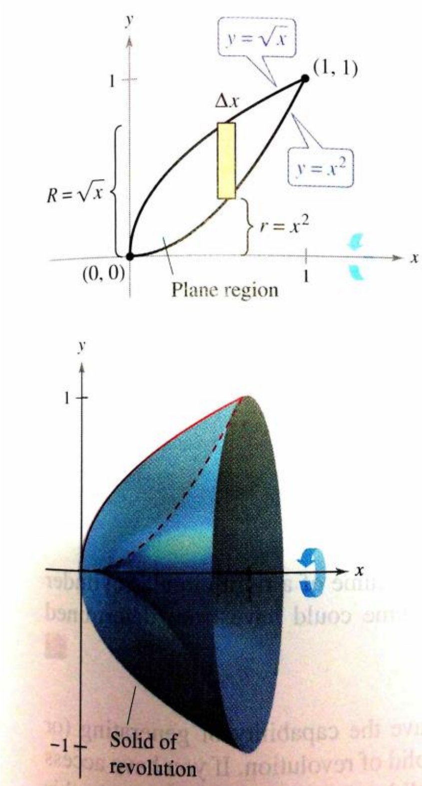
Figure 7.19

EXAMPLE 3 Using the Washer Method

Find the volume of the solid formed by revolving the region bounded by the graphs of

$$y = \sqrt{x} \quad \text{and} \quad y = x^2$$

about the x-axis, as shown in Figure 7.20.



Solid of revolution
Figure 7.20

compute the volume.

EXAMPLE 4 **Integrating with Respect to *y*, Two-Integral Case**

Find the volume of the solid formed by revolving the region bounded by the graphs of

$$y = x^2 + 1, \quad y = 0, \quad x = 0, \quad \text{and} \quad x = 1$$

about the y-axis, as shown in Figure 7.21.

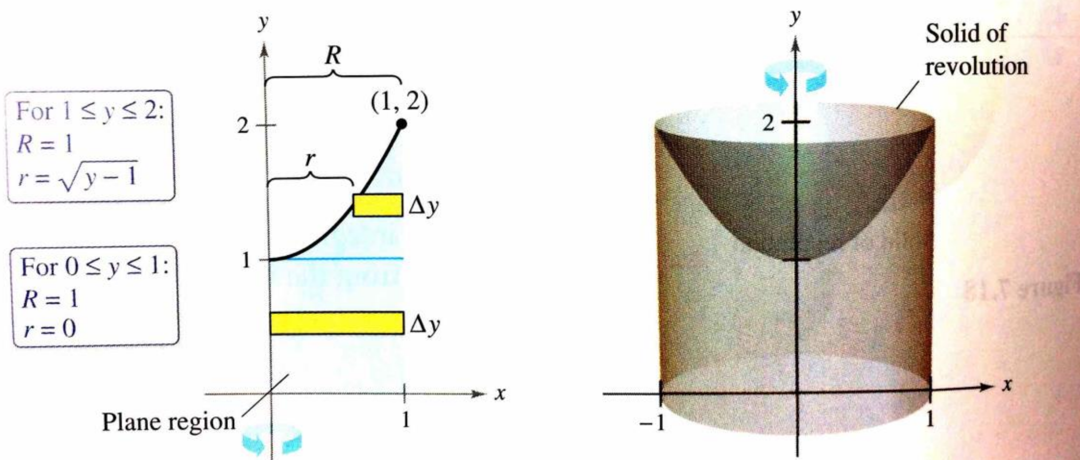


Figure 7.21

EXAMPLE 5 Manufacturing

⋮···▷ *See LarsonCalculus.com for an interactive version of this type of example.*

A manufacturer drills a hole through the center of a metal sphere of radius 5 inches, as shown in Figure 7.23(a). The hole has a radius of 3 inches. What is the volume of the resulting metal ring?

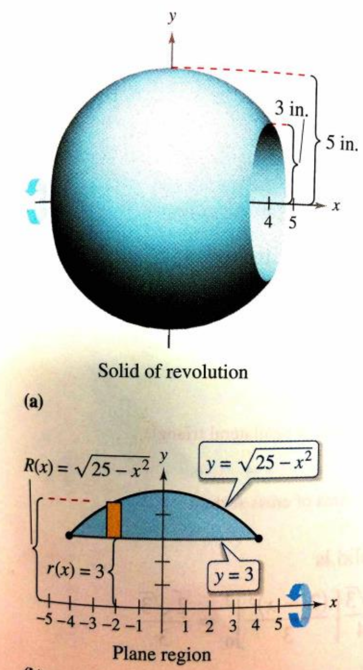


Figure 7.23

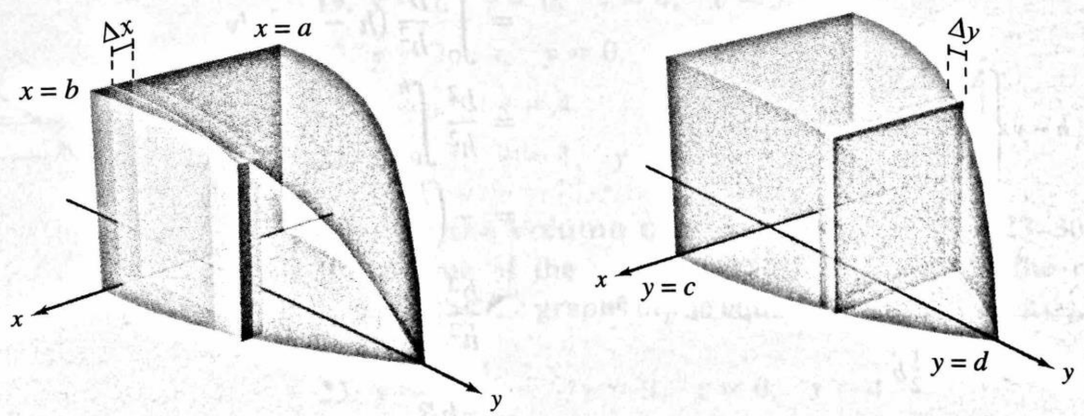
VOLUMES OF SOLIDS WITH KNOWN CROSS SECTIONS

1. For cross sections of area $A(x)$ taken perpendicular to the x-axis,

$$\text{Volume} = \int_a^b A(x)\,dx.$$ See Figure 7.24(a).

2. For cross sections of area $A(y)$ taken perpendicular to the y-axis,

$$\text{Volume} = \int_c^d A(y)\,dy.$$ See Figure 7.24(b).



(a) Cross sections perpendicular to x-axis **(b)** Cross sections perpendicular to y-axis

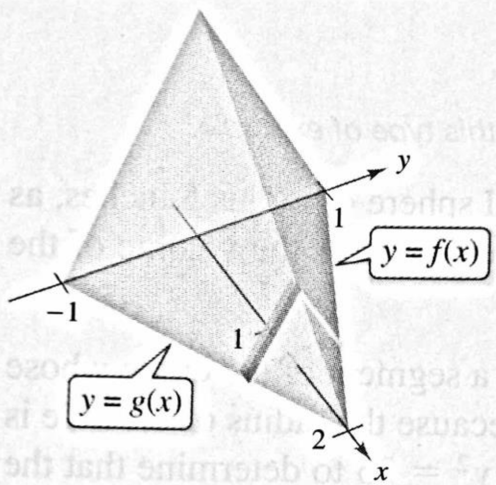
Figure 7.24

EXAMPLE 6 Triangular Cross Sections

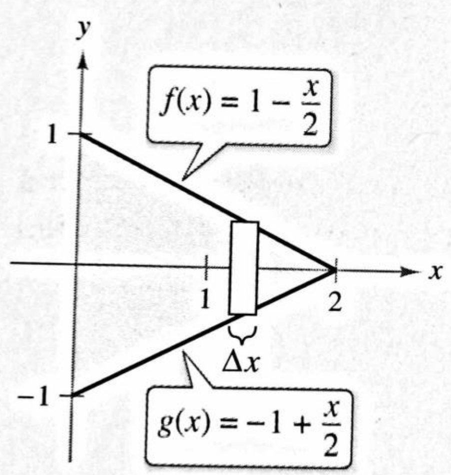
Find the volume of the solid shown in Figure 7.25. The base of the solid is the region bounded by the lines

$$f(x) = 1 - \frac{x}{2}, \quad g(x) = -1 + \frac{x}{2}, \quad \text{and} \quad x = 0.$$

The cross sections perpendicular to the x-axis are equilateral triangles.



Cross sections are equilateral triangles.



Triangular base in xy-plane
Figure 7.25

EXAMPLE 7 An Application to Geometry

Prove that the volume of a pyramid with a square base is

$$V = \frac{1}{3}hB$$

where h is the height of the pyramid and B is the area of the bas

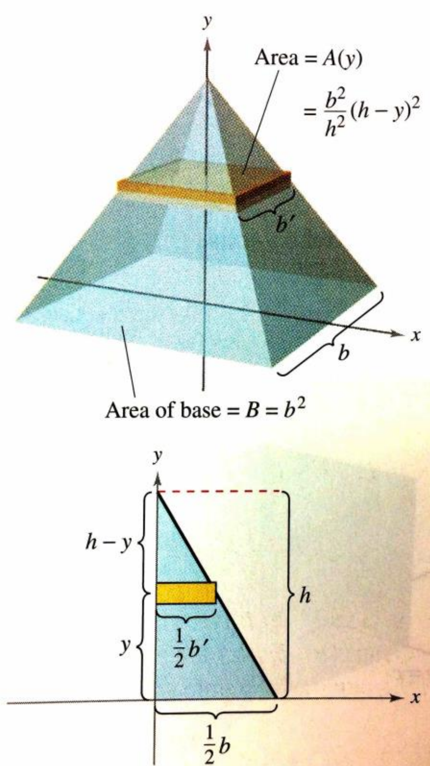


Figure 7.26