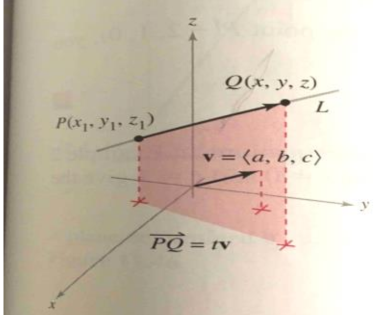


Line L and its direction vector v
Figure 11.43

THEOREM 11.11 Parametric Equations of a Line in Space

A line L parallel to the vector $\mathbf{v} = \langle a, b, c \rangle$ and passing through the point $P(x_1, y_1, z_1)$ is represented by the **parametric equations**

$$x = x_1 + at, \quad y = y_1 + bt, \quad \text{and} \quad z = z_1 + ct.$$

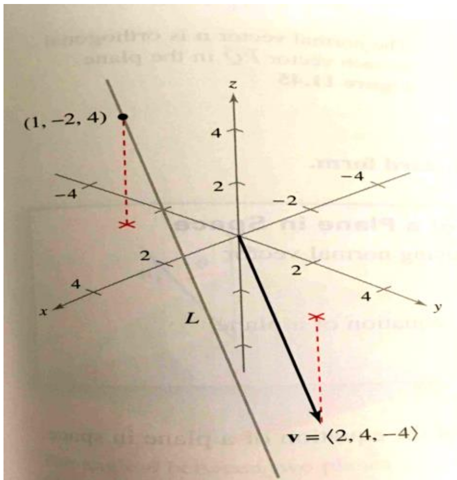
If the direction numbers a, b, and c are all nonzero, then you can eliminate the parameter t to obtain **symmetric equations** of the line.

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$$

Symmetric equations

EXAMPLE 1 Finding Parametric and Symmetric Equations

Find parametric and symmetric equations of the line L that passes through the point $(1, -2, 4)$ and is parallel to $\mathbf{v} = \langle 2, 4, -4 \rangle$, as shown in Figure 11.44.



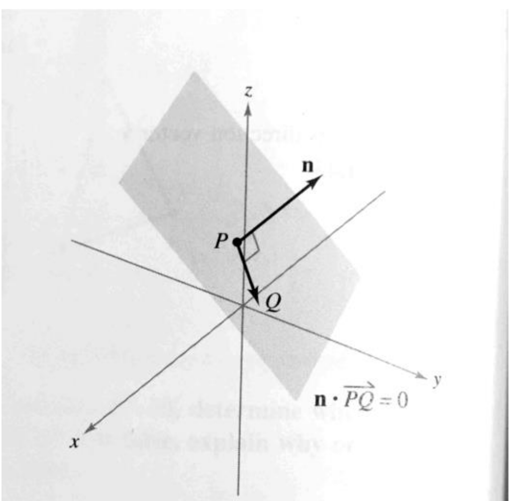
The vector **v** is parallel to the line L.
Figure 11.44

EXAMPLE 2 Parametric Equations of a Line Through Two Points

See LarsonCalculus.com for an interactive version of this

Find a set of parametric equations of the line that passes through the points

$$(-2, 1, 0) \quad \text{and} \quad (1, 3, 5).$$



The normal vector **n** is orthogonal to each vector $\overrightarrow{PQ}$ in the plane.
Figure 11.45

THEOREM 11.12 Standard Equation of a Plane in Space

The plane containing the point (x_1, y_1, z_1) and having normal vector

$$\mathbf{n} = \langle a, b, c \rangle$$

can be represented by the **standard form** of the equation of a plane

$$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0.$$

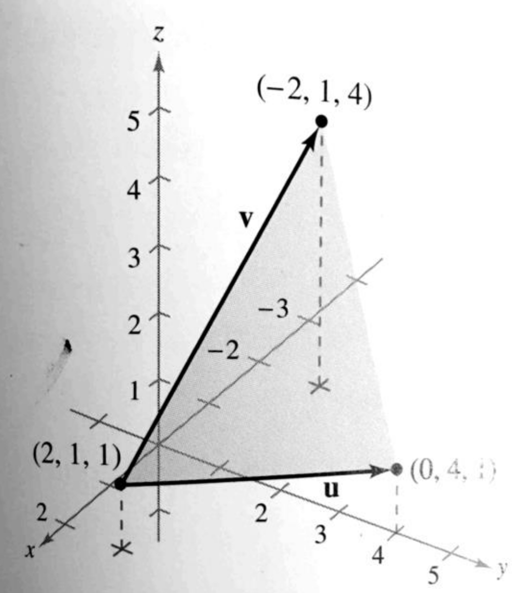
By regrouping terms, you obtain the **general form** of the equation of a plane in space.

$$ax + by + cz + d = 0 \qquad \text{General form of equation of plane}$$

EXAMPLE 3 Finding an Equation of a Plane in Three-Space

Find the general equation of the plane containing the points

$(2, 1, 1)$, $(0, 4, 1)$, and $(-2, 1, 4)$.

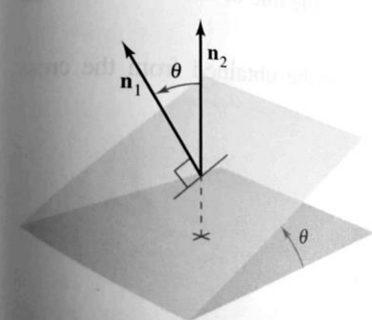


A plane determined by **u** and **v**
Figure 11.46

$$\cos \theta = \frac{|\mathbf{n}_1 \cdot \mathbf{n}_2|}{\|\mathbf{n}_1\| \, \|\mathbf{n}_2\|}.$$ Angle between two planes

Consequently, two planes with normal vectors $\mathbf{n}_1$ and $\mathbf{n}_2$ are

1. *perpendicular* when $\mathbf{n}_1 \cdot \mathbf{n}_2 = 0$.
2. *parallel* when $\mathbf{n}_1$ is a scalar multiple of $\mathbf{n}_2$.



The angle θ between two planes
Figure 11.47

EXAMPLE 4 Finding the Line of Intersection of Two Planes

Find the angle between the two planes

$$x - 2y + z = 0 \quad \text{and} \quad 2x + 3y - 2z = 0.$$

Then find parametric equations of their line of intersection (see Figure 11.48).

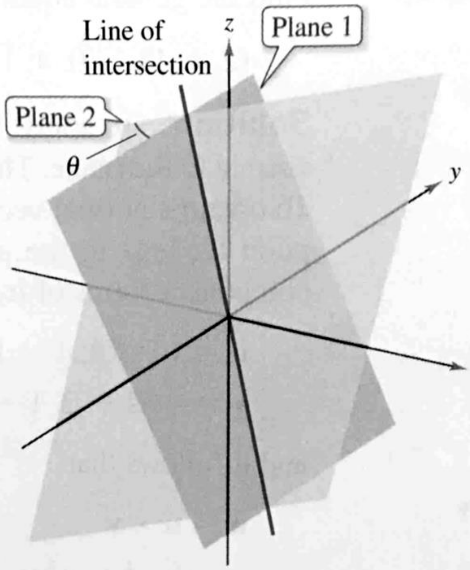
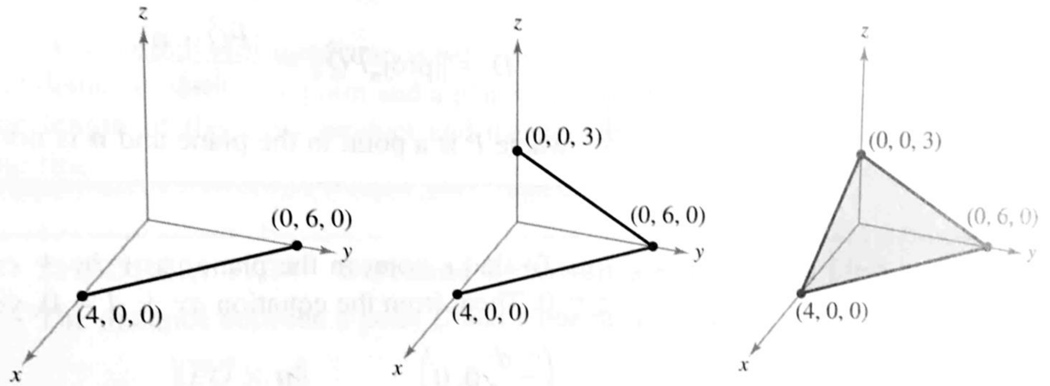


Figure 11.48



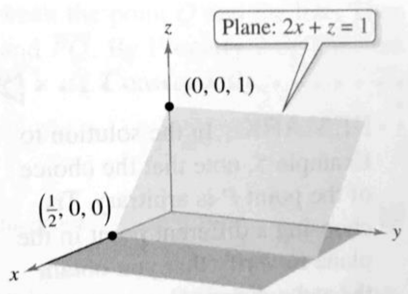
xy-trace ($z = 0$): $3x + 2y = 12$

yz-trace ($x = 0$): $2y + 4z = 12$

xz-trace ($y = 0$): $3x + 4z = 12$

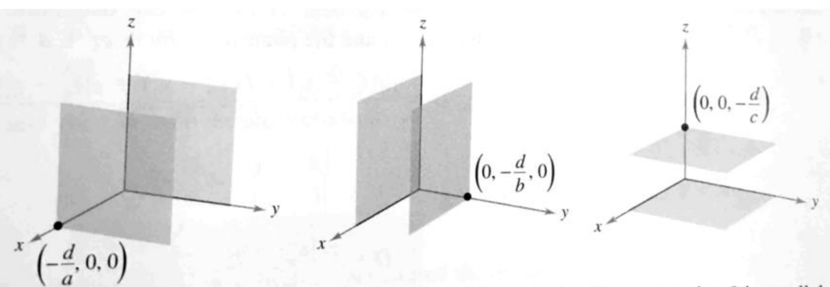
Traces of the plane $3x + 2y + 4z = 12$

Figure 11.49



Plane $2x + z = 1$ is parallel to the *y*-axis.

Figure 11.50

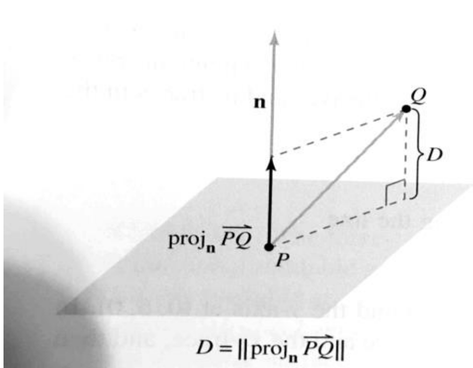


Plane $ax + d = 0$ is parallel to the *yz*-plane.

Plane $by + d = 0$ is parallel to the *xz*-plane.

Plane $cz + d = 0$ is parallel to the *xy*-plane.

Figure 11.51



The distance between a point and a plane
Figure 11.52

THEOREM 11.13 Distance Between a Point and a Plane

The distance between a plane and a point Q (not in the plane) is

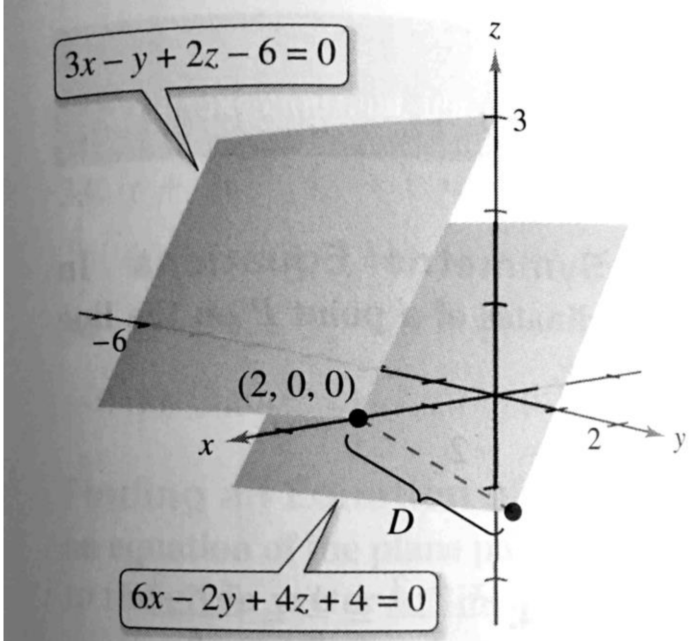
$$D = \|\text{proj}_{\mathbf{n}} \overrightarrow{PQ}\| = \frac{|\overrightarrow{PQ} \cdot \mathbf{n}|}{\|\mathbf{n}\|}$$

where P is a point in the plane and $\mathbf{n}$ is normal to the plane.

$$D = \frac{|ax_0 + by_0 + cz_0 + d|}{\sqrt{a^2 + b^2 + c^2}}$$

EXAMPLE 5 **Finding the Distance Between a Point and a Plane**

Find the distance between the point $Q(1, 5, -4)$ and the plane $3x - y + 2z = 6$.



The distance between the parallel planes is approximately 2.14.
Figure 11.53

EXAMPLE 6 Finding the Distance Between Two Parallel Planes

Two parallel planes, $3x - y + 2z - 6 = 0$ and $6x - 2y + 4z + 4 = 0$, are shown in Figure 11.53. To find the distance between the planes, choose a point in the first plane, such as $(x_0, y_0, z_0) = (2, 0, 0)$. Then, from the second plane, you can determine that $a = 6$, $b = -2$, $c = 4$, and $d = 4$, and conclude that the distance is

THEOREM 11.14 Distance Between a Point and a Line in Space

The distance between a point Q and a line in space is

$$D = \frac{\|\overrightarrow{PQ} \times \mathbf{u}\|}{\|\mathbf{u}\|}$$

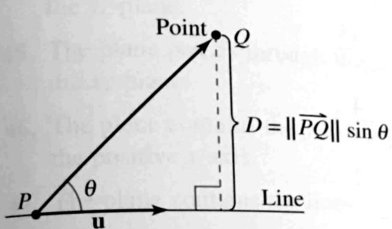
where $\mathbf{u}$ is a direction vector for the line and P is a point on the line.

a line

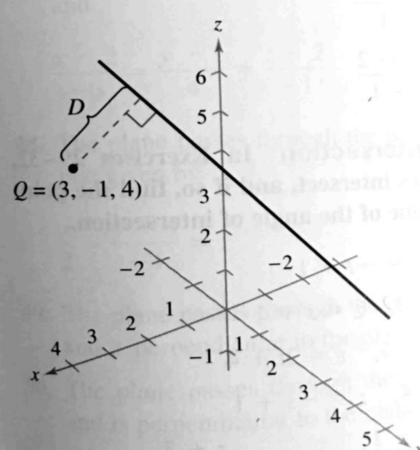
EXAMPLE 7 Finding the Distance Between a Point and a Line

Find the distance between the point $Q(3, -1, 4)$ and the line

$$x = -2 + 3t, \quad y = -2t, \quad \text{and} \quad z = 1 + 4t.$$



The distance between a point and a line
Figure 11.54



The distance between the point Q and the line is $\sqrt{6} \approx 2.45$.
Figure 11.55