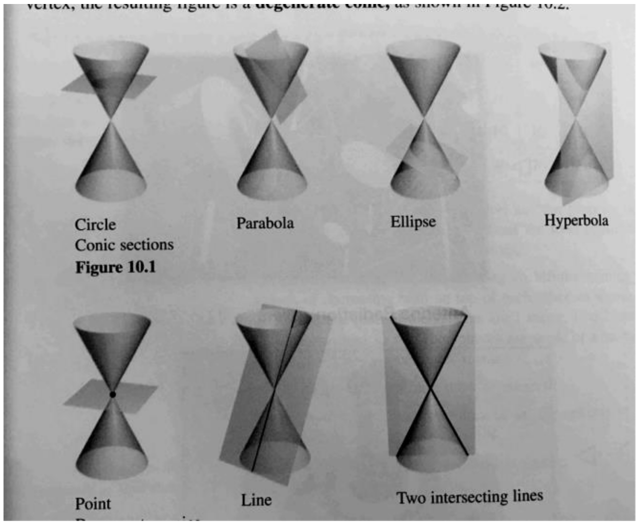


$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0.$$

General second-degree equation

$$(x - h)^2 + (y - k)^2 = r^2.$$

Standard equation of a circle

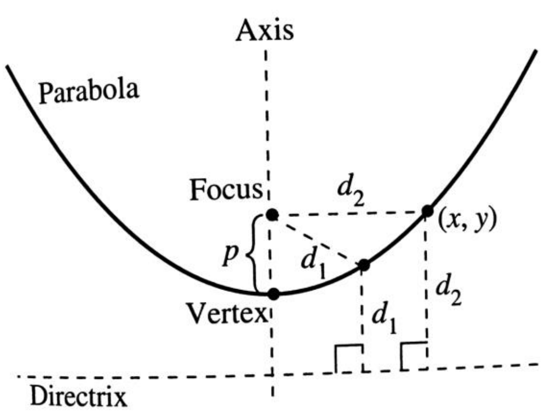


Figure 10.3

THEOREM 10.1 Standard Equation of a Parabola

The **standard form** of the equation of a parabola with vertex (h, k) and directrix $y = k - p$ is

$$(x - h)^2 = 4p(y - k). \qquad \text{Vertical axis}$$

For directrix $x = h - p$, the equation is

$$(y - k)^2 = 4p(x - h). \qquad \text{Horizontal axis}$$

The focus lies on the axis p units (*directed distance*) from the vertex. The coordinates of the focus are as follows.

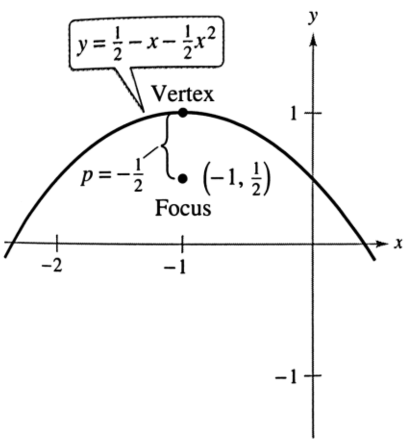
$(h, k + p)$ Vertical axis

$(h + p, k)$ Horizontal axis

EXAMPLE 1 Finding the Focus of a Parabola

Find the focus of the parabola

$$y = \frac{1}{2} - x - \frac{1}{2}x^2.$$



Parabola with a vertical axis, $p < 0$
Figure 10.4

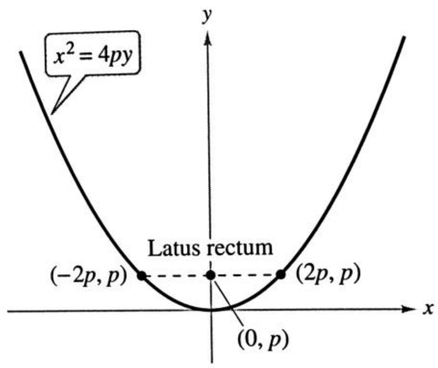
EXAMPLE 2 Focal Chord Length and Arc Length

See LarsonCalculus.com for an interactive version of this type of exam

Find the length of the latus rectum of the parabola

$$x^2 = 4py.$$

Then find the length of the parabolic arc intercepted by the latus rectum.

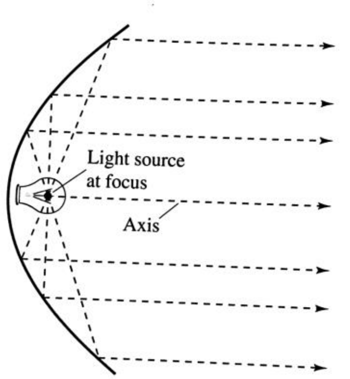


Length of latus rectum: $4p$
Figure 10.5

THEOREM 10.2 Reflective Property of a Parabola

Let P be a point on a parabola. The tangent line to the parabola at point P makes equal angles with the following two lines.

1. The line passing through P and the focus
2. The line passing through P parallel to the axis of the parabola



Parabolic reflector: light is reflected in parallel rays.
Figure 10.6

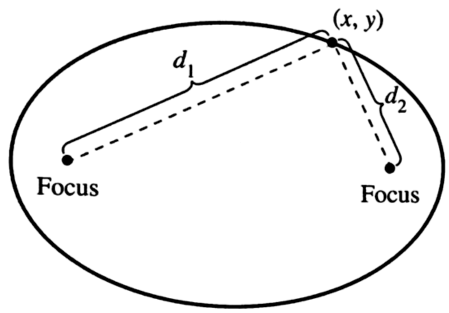


Figure 10.7

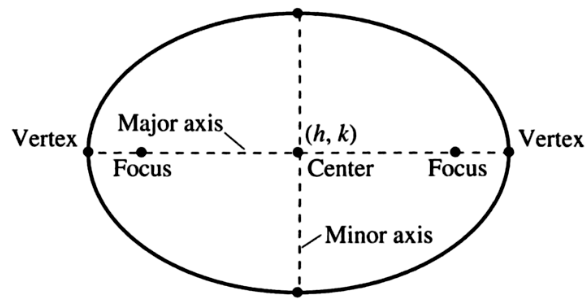


Figure 10.8

THEOREM 10.3 Standard Equation of an Ellipse

The standard form of the equation of an ellipse with center (h, k) and major and minor axes of lengths $2a$ and $2b$, where $a > b$, is

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1 \qquad \text{Major axis is horizontal.}$$

or

$$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1. \qquad \text{Major axis is vertical.}$$

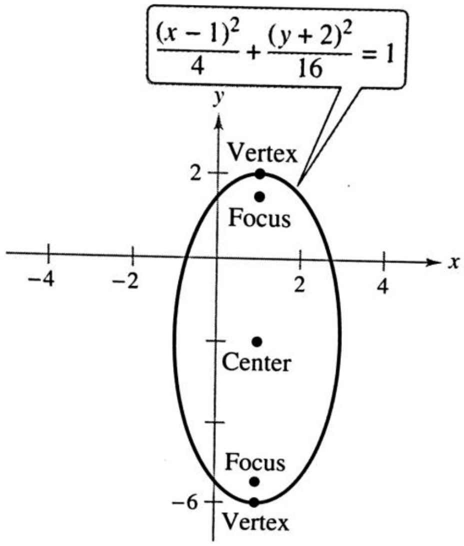
The foci lie on the major axis, c units from the center, with

$$c^2 = a^2 - b^2.$$

EXAMPLE 3 Analyzing an Ellipse

Find the center, vertices, and foci of the ellipse

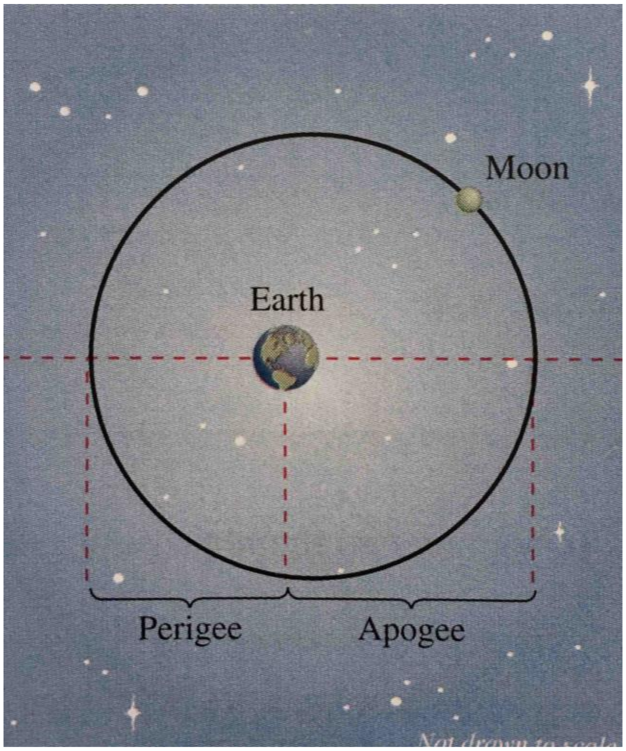
$$4x^2 + y^2 - 8x + 4y - 8 = 0.$$



Ellipse with a vertical major axis.
Figure 10.10

EXAMPLE 4 The Orbit of the Moon

The moon orbits Earth in an elliptical path with the center of Earth at one focus, as shown in Figure 10.11. The major and minor axes of the orbit have lengths of 768,800 kilometers and 767,640 kilometers, respectively. Find the greatest and least distances (the apogee and perigee) from Earth's center to the moon's center.



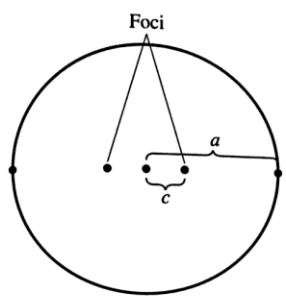
THEOREM 10.4 Reflective Property of an Ellipse

Let P be a point on an ellipse. The tangent line to the ellipse at point P makes equal angles with the lines through P and the foci.

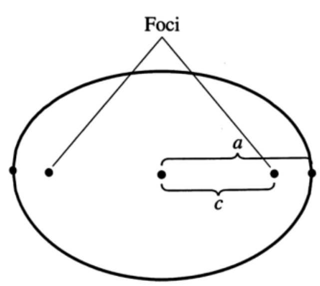
Definition of Eccentricity of an Ellipse

The **eccentricity** e of an ellipse is given by the ratio

$$e = \frac{c}{a}.$$



(a) $\frac{c}{a}$ is small.



(b) $\frac{c}{a}$ is close to 1.

Eccentricity is the ratio $\frac{c}{a}$.

Figure 10.12

EXAMPLE 5 Finding the Circumference of an Ellipse

See LarsonCalculus.com for an interactive version of this type of example.

Show that the circumference of the ellipse $(x^2/a^2) + (y^2/b^2) = 1$ is

$$4a\int_0^{\pi/2} \sqrt{1 - e^2 \sin^2 \theta}\, d\theta. \qquad e = \frac{c}{a}$$

EXAMPLE 6 Approximating the Value of an Elliptic Integral

Use the elliptic integral in Example 5 to approximate the circumference of the ellipse

$$\frac{x^2}{25} + \frac{y^2}{16} = 1.$$

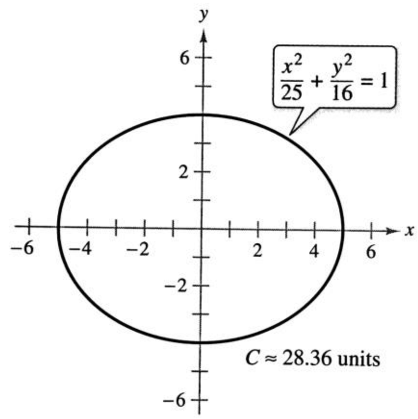


Figure 10.13

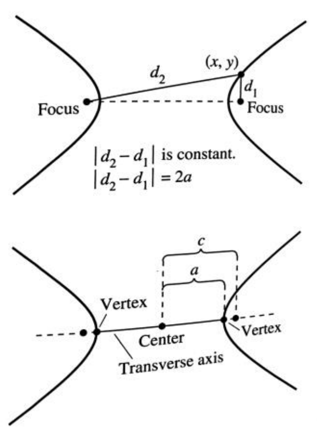


Figure 10.14

THEOREM 10.5 Standard Equation of a Hyperbola

The standard form of the equation of a hyperbola with center at (h, k) is

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1 \qquad \text{Transverse axis is horizontal.}$$

or

$$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1. \qquad \text{Transverse axis is vertical.}$$

The vertices are a units from the center, and the foci are c units from the center, where $c^2 = a^2 + b^2$.

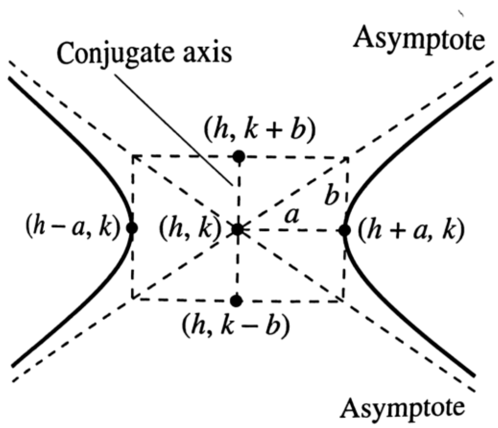


Figure 10.15

THEOREM 10.6 Asymptotes of a Hyperbola

For a *horizontal* transverse axis, the equations of the asymptotes are

$$y = k + \frac{b}{a}(x - h) \quad \text{and} \quad y = k - \frac{b}{a}(x - h).$$

For a *vertical* transverse axis, the equations of the asymptotes are

$$y = k + \frac{a}{b}(x - h) \quad \text{and} \quad y = k - \frac{a}{b}(x - h).$$

EXAMPLE 7 Using Asymptotes to Sketch a Hyperbola

See LarsonCalculus.com for an interactive version of this type of example.

Sketch the graph of the hyperbola

$$4x^2 - y^2 = 16.$$

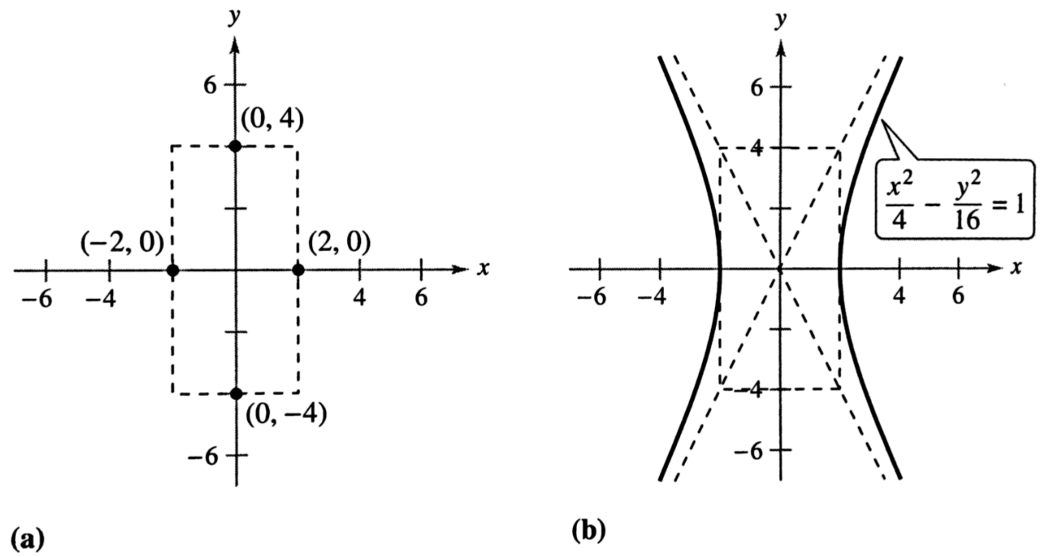


Figure 10.16

Definition of Eccentricity of a Hyperbola

The **eccentricity** e of a hyperbola is given by the ratio

$$e = \frac{c}{a}.$$

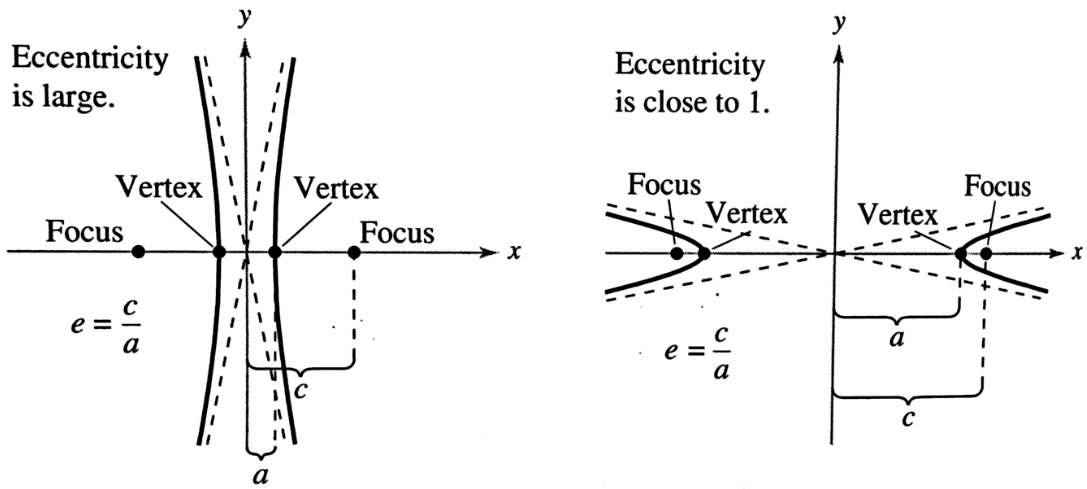
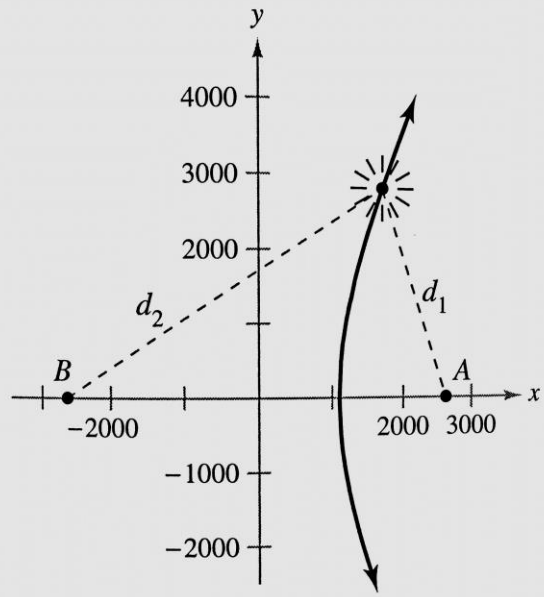


Figure 10.17

EXAMPLE 8 **A Hyperbolic Detection System**

Two microphones, 1 mile apart, record an explosion. Microphone *A* receives the sound 2 seconds before microphone *B*. Where was the explosion?



$2c = 5280$

$d_2 - d_1 = 2a = 2200$

Figure 10.18