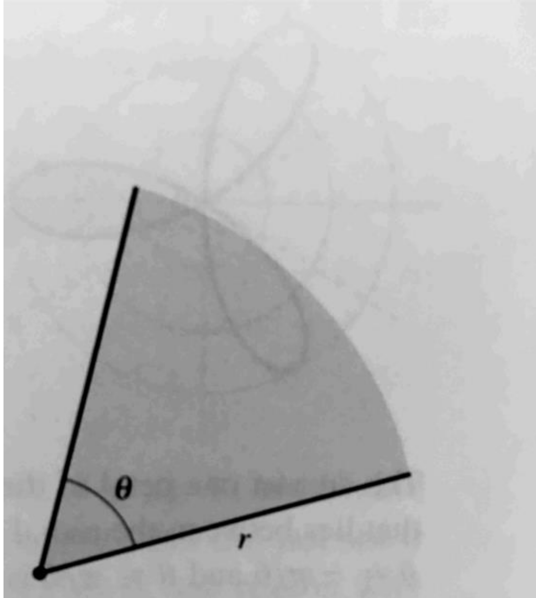


The area of a sector of a circle is $A = \frac{1}{2}\theta r^2$.
Figure 10.48

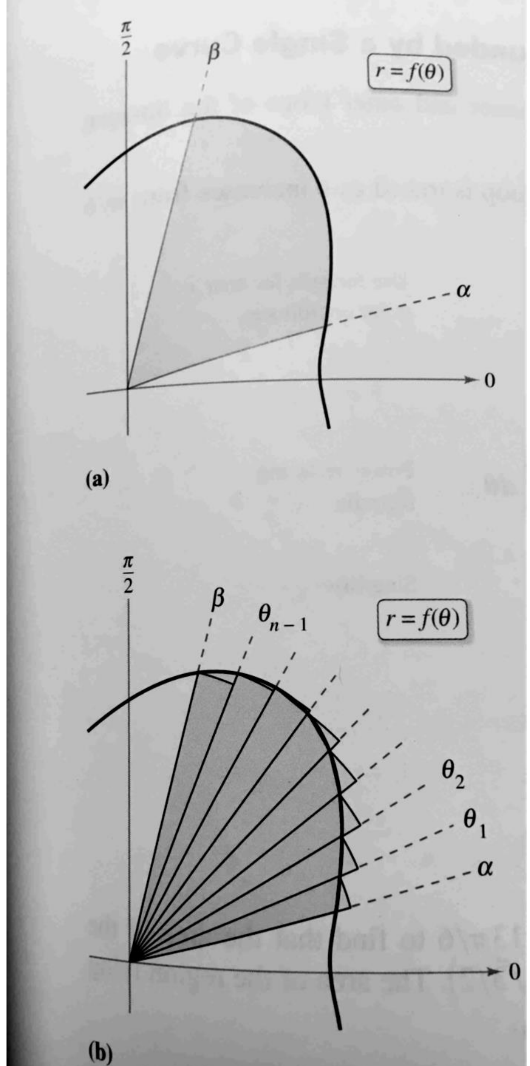


Figure 10.49

THEOREM 10.13 Area in Polar Coordinates

If f is continuous and nonnegative on the interval $[\alpha, \beta]$, $0 < \beta - \alpha \le 2\pi$, then the area of the region bounded by the graph of $r = f(\theta)$ between the radial lines $\theta = \alpha$ and $\theta = \beta$ is

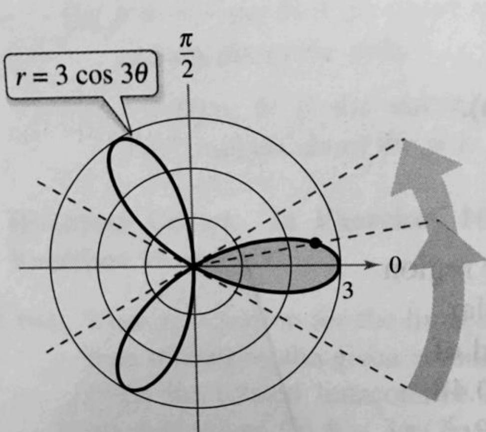
$$A = \frac{1}{2}\int_{\alpha}^{\beta} [f(\theta)]^2 \, d\theta$$

$$= \frac{1}{2}\int_{\alpha}^{\beta} r^2 \, d\theta. \qquad 0 < \beta - \alpha \le 2\pi$$

EXAMPLE 1 Finding the Area of a Polar Region

See LarsonCalculus.com for an interactive version of this type of example.

Find the area of one petal of the rose curve $r = 3 \cos 3\theta$.

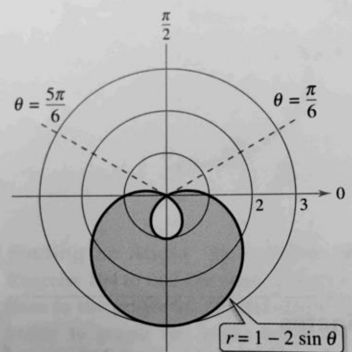


The area of one petal of the rose curve that lies between the radial lines $\theta = -\pi/6$ and $\theta = \pi/6$ is $3\pi/4$.

Figure 10.50

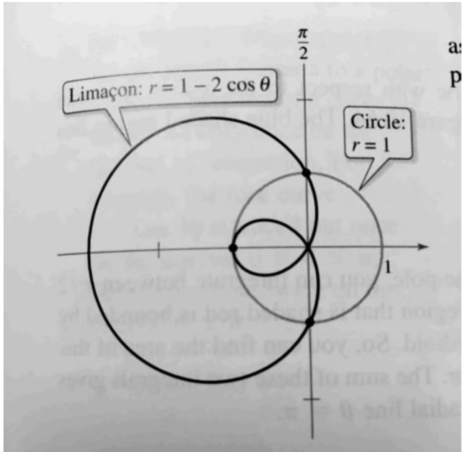
EXAMPLE 2 **Finding the Area Bounded by a Single Curve**

Find the area of the region lying between the inner and outer loops of the limaçon $r = 1 - 2\sin\theta$.



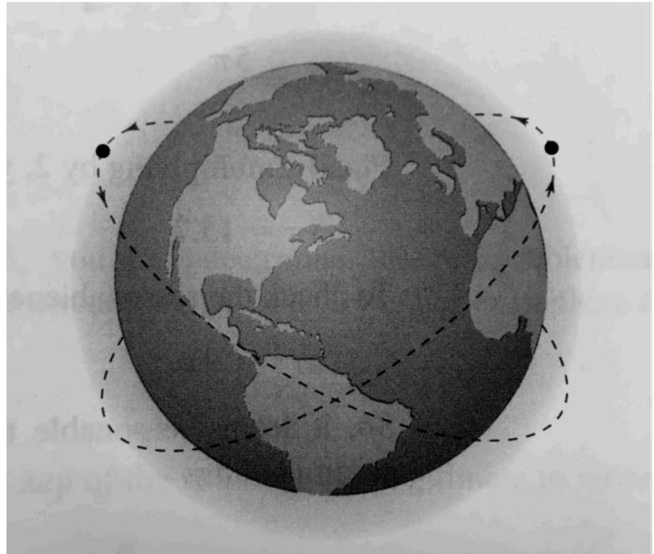
The area between the inner and outer loops is approximately 8.34.

Figure 10.51



Three points of intersection: $(1, \pi/2)$, $(-1, 0)$, and $(1, 3\pi/2)$

Figure 10.52



The paths of satellites can cross without causing a collision.

Figure 10.53

EXAMPLE 3 **Finding the Area of a Region Between Two Curves**

Find the area of the region common to the two regions bounded by the curves.

$r = -6 \cos \theta$ Circle

and

$r = 2 - 2 \cos \theta.$ Cardioid

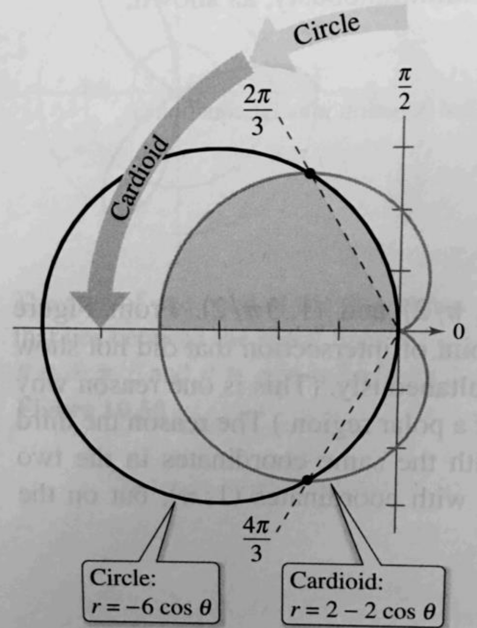


Figure 10.54

THEOREM 10.14 Arc Length of a Polar Curve

Let f be a function whose derivative is continuous on an interval $\alpha \le \theta \le \beta$. The length of the graph of $r = f(\theta)$ from $\theta = \alpha$ to $\theta = \beta$ is

$$s = \int_{\alpha}^{\beta} \sqrt{[f(\theta)]^2 + [f'(\theta)]^2}\, d\theta = \int_{\alpha}^{\beta} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta.$$

EXAMPLE 4 **Finding the Length of a Polar Curve**

Find the length of the arc from $\theta = 0$ to $\theta = 2\pi$ for the cardioid

$$r = f(\theta) = 2 - 2\cos\theta$$

as shown in Figure 10.55.

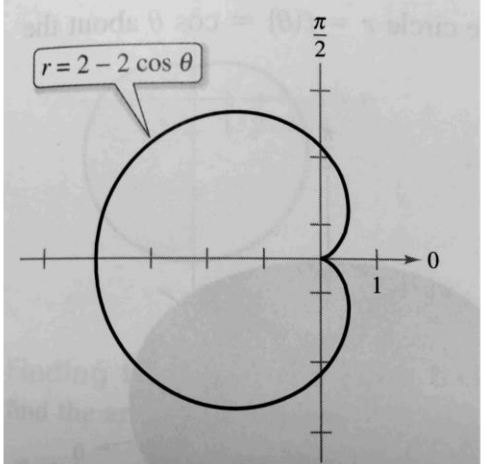


Figure 10.55

THEOREM 10.15 Area of a Surface of Revolution

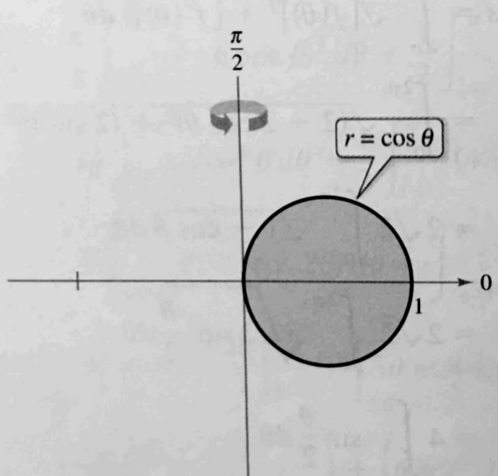
Let f be a function whose derivative is continuous on an interval $\alpha \le \theta \le \beta$. The area of the surface formed by revolving the graph of $r = f(\theta)$ from $\theta = \alpha$ to $\theta = \beta$ about the indicated line is as follows.

1. $S = 2\pi \int_{\alpha}^{\beta} f(\theta) \sin\theta \sqrt{[f(\theta)]^2 + [f'(\theta)]^2}\, d\theta$ About the polar axis

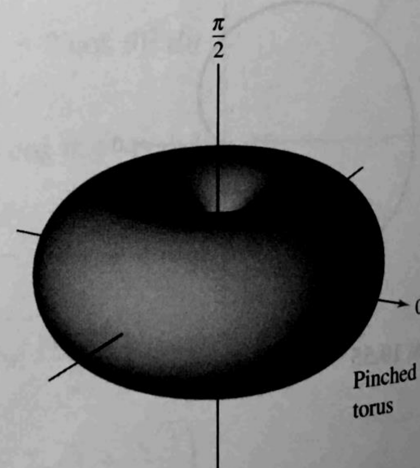
2. $S = 2\pi \int_{\alpha}^{\beta} f(\theta) \cos\theta \sqrt{[f(\theta)]^2 + [f'(\theta)]^2}\, d\theta$ About the line $\theta = \frac{\pi}{2}$

EXAMPLE 5 **Finding the Area of a Surface of Revolution**

Find the area of the surface formed by revolving the circle $r = f(\theta) = \cos\theta$ about the line $\theta = \pi/2$, as shown in Figure 10.56.



(a)



(b)

Figure 10.56