EXAMPLE 8 Matrices and Digital Photography

The letter L in **Figure 8.7** is shown using 9 pixels in a 3×3 grid. The colors possible in the grid are shown in **Figure 8.8**. Each color is represented by a specific number: 0, 1, 2, or 3.

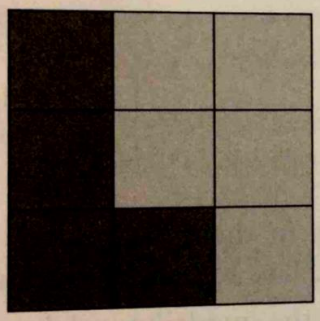


FIGURE 8.7 The letter L

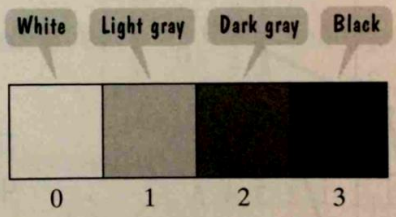


FIGURE 8.8 Color levels

a. Find a matrix that represents a digital photograph of this letter L.

b. Increase the contrast of the letter L by changing the dark gray to black and the light gray to white. Use matrix addition to accomplish this.

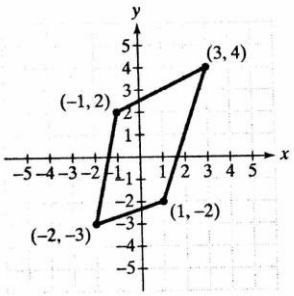


FIGURE 8.10

EXAMPLE 9 Transformations of an Image

The quadrilateral in **Figure 8.10** can be represented by the matrix

$$A = \begin{bmatrix} -2 & -1 & 3 & 1 \\ -3 & 2 & 4 & -2 \end{bmatrix}$$

Coordinates of vertices; x-coordinates; y-coordinates

Each column in the matrix gives the coordinates of a vertex, or corner, of the quadrilateral. Use matrix operations to perform the following transformations:

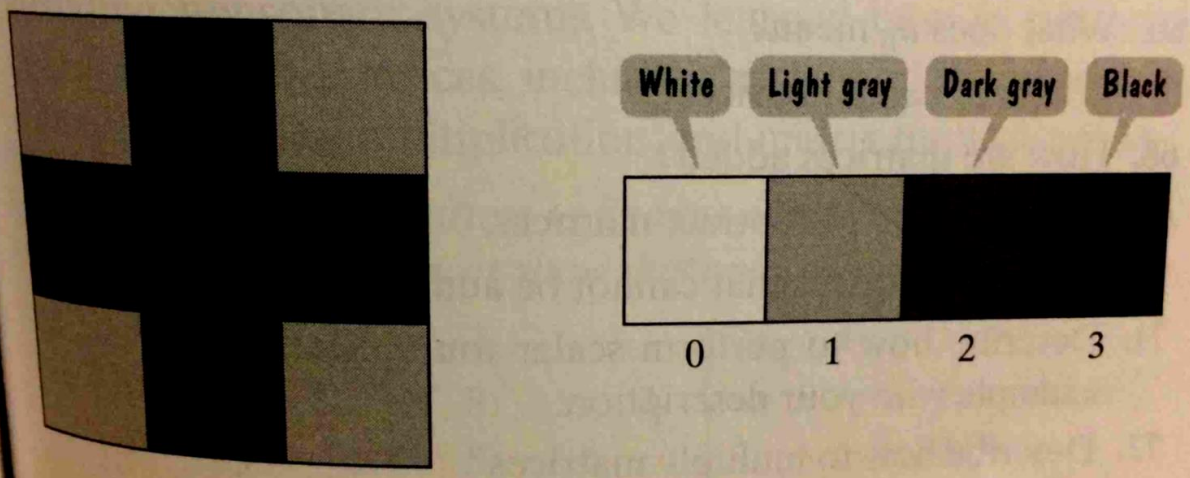
a. Move the quadrilateral 4 units to the right and 1 unit down.

b. Shrink the quadrilateral to half its perimeter.

c. Let $B = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$. Find BA. What effect does this have on the quadrilateral in **Figure 8.10**?

Application Exercises

The + sign in the figure is shown using 9 pixels in a 3 × 3 grid. The color levels are given to the right of the figure. Each color is represented by a specific number: 0, 1, 2, or 3. Use this information to solve Exercises 51–52.



51. **a.** Find a matrix that represents a digital photograph of the + sign. *

b. Adjust the contrast by changing the black to dark gray and the light gray to white. Use matrix addition to accomplish this. *

c. Adjust the contrast by changing the black to light gray and the light gray to dark gray. Use matrix addition to accomplish this. *

In Exercises 6–10, perform the indicated matrix operations or solve the matrix equation for X given that A, B, and C are defined as follows. If an operation is not defined, state the reason.

$$A = \begin{bmatrix} 0 & 2 \\ -1 & 3 \\ 1 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 4 & 1 \\ -6 & -2 \end{bmatrix} \quad C = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

6. $2C - \frac{1}{2}B$ *

7. $A(B + C)$ *

8. $A(BC)$ *

9. $A + C$ *

10. $2X - 3C = B$ *

Application Exercises

In Exercises 51–52, use the coding matrix

$$A = \begin{bmatrix} 4 & -1 \\ -3 & 1 \end{bmatrix} \text{ and its inverse } A^{-1} = \begin{bmatrix} 1 & 1 \\ 3 & 4 \end{bmatrix}$$

to encode and then decode the given message.

51. HELP *

52. LOVE *

In Exercises 53–54, use the coding matrix

$$A = \begin{bmatrix} 1 & -1 & 0 \\ 3 & 0 & 2 \\ -1 & 0 & -1 \end{bmatrix} \text{ and its inverse}$$

$$A^{-1} = \begin{bmatrix} 0 & 1 & 2 \\ -1 & 1 & 2 \\ 0 & -1 & -3 \end{bmatrix} \text{ to write a cryptogram for each}$$

message. Check your result by decoding the cryptogram.

53.

S	E	N	D	_	C	A	S	H
19	5	14	4	0	3	1	19	8

$$\text{Use } \begin{bmatrix} 19 & 4 & 1 \\ 5 & 0 & 19 \\ 14 & 3 & 8 \end{bmatrix}. \quad *$$

54.

S	T	A	Y	_	W	E	L	L
19	20	1	25	0	23	5	12	12

$$\text{Use } \begin{bmatrix} 19 & 25 & 5 \\ 20 & 0 & 12 \\ 1 & 23 & 12 \end{bmatrix}. \quad *$$