EXAMPLE 1 **Using a Tangent Line Approximation**

See LarsonCalculus.com for an interactive version of this type of example.

Find the tangent line approximation of $f(x) = 1 + \sin x$ at the point $(0, 1)$. Then use a table to compare the y-values of the linear function with those of $f(x)$ on an open interval containing $x = 0$.

Definition of Differentials

Let $y = f(x)$ represent a function that is differentiable on an open interval containing x. The **differential of x** (denoted by dx) is any nonzero real number. The **differential of y** (denoted by dy) is

$$dy = f'(x)\,dx.$$

EXAMPLE 2 Comparing Δy and dy

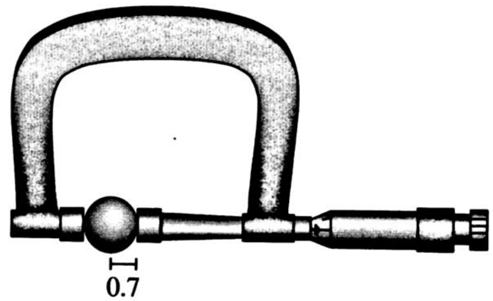
Let $y = x^2$. Find dy when $x = 1$ and $dx = 0.01$. Compare this value with Δy for $x = 1$ and $\Delta x = 0.01$.

Comparing Δy and dy In Exercises 7–10, use the information to evaluate and compare Δy and dy.

Function	x-Value	Differential of x
7. $y = x^3$	$x = 1$	$\Delta x = dx = 0.1$
8. $y = 6 - 2x^2$	$x = -2$	$\Delta x = dx = 0.1$

EXAMPLE 3 Estimation of Error

The measured radius of a ball bearing is 0.7 inch, as shown in the figure. The measurement is correct to within 0.01 inch. Estimate the propagated error in the volume V of the ball bearing.



Ball bearing with measured radius that is correct to within 0.01 inch.

Solution The formula for the volume of a sphere is

$$V = \frac{4}{3}\pi r^3$$

Differential Formulas

Let u and v be differentiable functions of x.

Constant multiple: $d[cu] = c\,du$

Sum or difference: $d[u \pm v] = du \pm dv$

Product: $d[uv] = u\,dv + v\,du$

Quotient: $d\left[\dfrac{u}{v}\right] = \dfrac{v\,du - u\,dv}{v^2}$

EXAMPLE 4

Function

a. $y = x^2$

b. $y = \sqrt{x}$

c. $y = 2 \sin x$

d. $y = x \cos x$

e. $y = \dfrac{1}{x}$

EXAMPLE 5 Finding the Differential of a Composite Function

$y = f(x) = \sin 3x$ Original function

EXAMPLE 6 Finding the Differential of a Composite Function

$y = f(x) = (x^2 + 1)^{1/2}$ Original function

> $$f(x + \Delta x) \approx f(x) + dy = f(x) + f'(x)\,dx$$

which is derived from the approximation

$$\Delta y = f(x + \Delta x) - f(x) \approx dy.$$

The key to using this formula is to choose a value for x that makes the easier, as shown in Example 7.

EXAMPLE 7 Approximating Function Values

Use differentials to approximate $\sqrt{16.5}$.

31. Stopping Distance The total stopping distance T of a vehicle is

$$T = 2.5x + 0.5x^2$$

where T is in feet and x is the speed in miles per hour. Approximate the change and percent change in total stopping distance as speed changes from $x = 25$ to $x = 26$ miles per hour.

36. **Surveying** A surveyor standing 50 feet from the base of a large tree measures the angle of elevation to the top of the tree as 71.5°. How accurately must the angle be measured if the percent error in estimating the height of the tree is to be less than 6%?