Implicit Differentiation

GUIDELINES FOR IMPLICIT DIFFERENTIATION

1. Differentiate both sides of the equation *with respect to x*.
2. Collect all terms involving dy/dx on the left side of the equation and move all other terms to the right side of the equation.
3. Factor dy/dx out of the left side of the equation.
4. Solve for dy/dx.

EXAMPLE 2 **Implicit Differentiation**

Find dy/dx given that $y^3 + y^2 - 5y - x^2 = -4$.

EXAMPLE 3 **Graphs and Differentiable Functions**

If possible, represent y as a differentiable function of x.

a. $x^2 + y^2 = 0$ **b.** $x^2 + y^2 = 1$ **c.** $x + y^2 = 1$

EXAMPLE 4 **Finding the Slope of a Graph Implicitly**

See LarsonCalculus.com for an interactive version of this type of example.

Determine the slope of the tangent line to the graph of $x^2 + 4y^2 = 4$ at the point $\left(\sqrt{2}, -1/\sqrt{2}\right)$. See Figure 2.29.

EXAMPLE 5 **Finding the Slope of a Graph Implicitly**

Determine the slope of the graph of

$$3(x^2 + y^2)^2 = 100xy$$

at the point $(3, 1)$.

EXAMPLE 6 **Determining a Differentiable Function**

Find dy/dx implicitly for the equation $\sin y = x$. Then find the largest interval of the form $-a < y < a$ on which y is a differentiable function of x (see Figure 2.31).

EXAMPLE 7 **Finding the Second Derivative Implicitly**

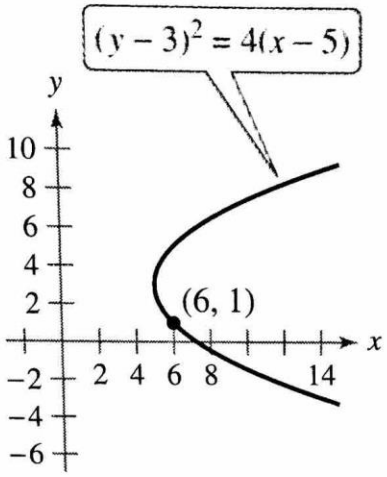
Given $x^2 + y^2 = 25$, find $\dfrac{d^2y}{dx^2}$.

EXAMPLE 8 **Finding a Tangent Line to a Graph**

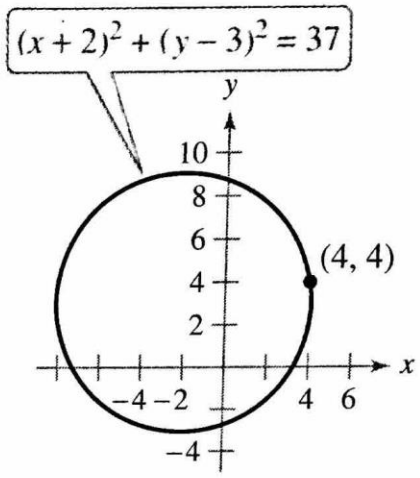
Find the tangent line to the graph of $x^2(x^2 + y^2) = y^2$ at the point $\left(\sqrt{2}/2, \sqrt{2}/2\right)$, as shown in Figure 2.32.

Famous Curves **In Exercises 33–40, find an equation of the tangent line to the graph at the given point. To print an enlarged copy of the graph, go to *MathGraphs.com*.**

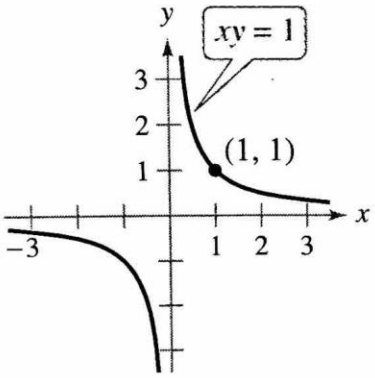
33. Parabola



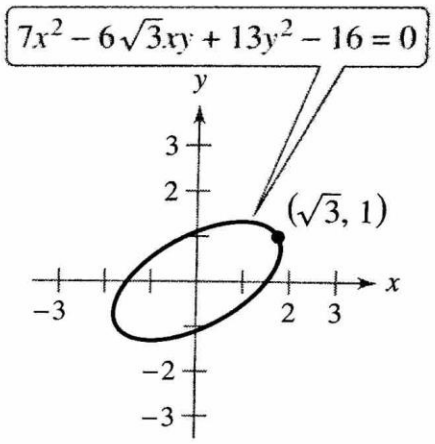
34. Circle



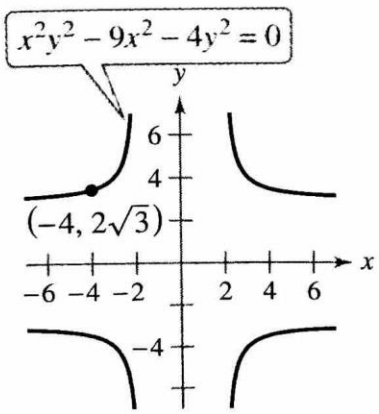
35. Rotated hyperbola



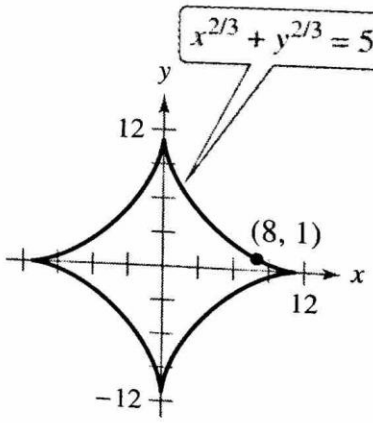
36. Rotated ellipse



37. Cruciform



38. Astroid



33. $(y-3)^2 = 4(x-5), \quad (6,1)$

$$2(y-3)y' = 4$$

$$y' = \frac{2}{y-3}$$

At $(6,1)$: $y' = \frac{2}{1-3} = -1$

Tangent line: $y - 1 = -1(x-6)$

$$y = -x + 7$$

34. $(x+2)^2 + (y-3)^2 = 37, \quad (4,4)$

$$2(x+2) + 2(y-3)y' = 0$$

$$(y-3)y' = -(x+2)$$

$$y' = -\frac{(x+2)}{y-3}$$

At $(4,4)$: $y' = -\frac{6}{1} = -6$

Tangent line: $y - 4 = -6(x-4)$

$$y = -6x + 28$$

35. $xy = 1, \quad (1, 1)$

$$xy' + y = 0$$

$$y' = \frac{-y}{x}$$

At $(1, 1)$: $y' = -1$

Tangent line: $y - 1 = -1(x - 1)$

$$y = -x + 2$$

36. $7x^2 - 6\sqrt{3}xy + 13y^2 - 16 = 0, \quad (\sqrt{3}, 1)$

$$14x - 6\sqrt{3}xy' - 6\sqrt{3}y + 26yy' = 0$$

$$y' = \frac{6\sqrt{3}y - 14x}{26y - 6\sqrt{3}x}$$

At $(\sqrt{3}, 1)$: $y' = \dfrac{6\sqrt{3} - 14\sqrt{3}}{26 - 6\sqrt{3}\sqrt{3}} = \dfrac{-8\sqrt{3}}{8} = -\sqrt{3}$

Tangent line: $y - 1 = -\sqrt{3}(x - \sqrt{3})$

$$y = -\sqrt{3}x + 4$$

37. $x^2y^2 - 9x^2 - 4y^2 = 0, \quad \left(-4, 2\sqrt{3}\right)$

$$x^2 2yy' + 2xy^2 - 18x - 8yy' = 0$$

$$y' = \frac{18x - 2xy^2}{2x^2y - 8y}$$

At $\left(-4, 2\sqrt{3}\right)$: $y' = \dfrac{18(-4) - 2(-4)(12)}{2(16)\left(2\sqrt{3}\right) - 16\sqrt{3}}$

$$= \frac{24}{48\sqrt{3}} = \frac{1}{2\sqrt{3}} = \frac{\sqrt{3}}{6}$$

Tangent line: $y - 2\sqrt{3} = \dfrac{\sqrt{3}}{6}(x + 4)$

$$y = \frac{\sqrt{3}}{6}x + \frac{8}{3}\sqrt{3}$$

38. $x^{2/3} + y^{2/3} = 5, \quad (8, 1)$

$$\frac{2}{3}x^{-1/3} + \frac{2}{3}y^{-1/3}y' = 0$$

$$y' = \frac{-x^{-1/3}}{y^{-1/3}} = -\left(\frac{y}{x}\right)^{1/3}$$

At $(8, 1)$: $y' = -\dfrac{1}{2}$

Tangent line: $y - 1 = -\dfrac{1}{2}(x - 8)$

$$y = -\frac{1}{2}x + 5$$

Algebra:

$$6(x^2+y^2)\left(2x+2y\frac{dy}{dx}\right)=100\left(x\frac{dy}{dx}+y\right)$$

$$6\left(2x^3+2x^2y\frac{dy}{dx}+2xy^2+2y^3\frac{dy}{dx}\right)=100x\frac{dy}{dx}+100y$$

$$12x^3+12x^2y\frac{dy}{dx}+12xy^2+12y^3\frac{dy}{dx}=100x\frac{dy}{dx}+100y$$

$$12x^2y\frac{dy}{dx}+12y^3\frac{dy}{dx}=100x\frac{dy}{dx}+100y-12x^3-12xy^2$$

$$12y(x^2+y^2)\frac{dy}{dx}=100x\frac{dy}{dx}+100y-12x(x^2+y^2)$$

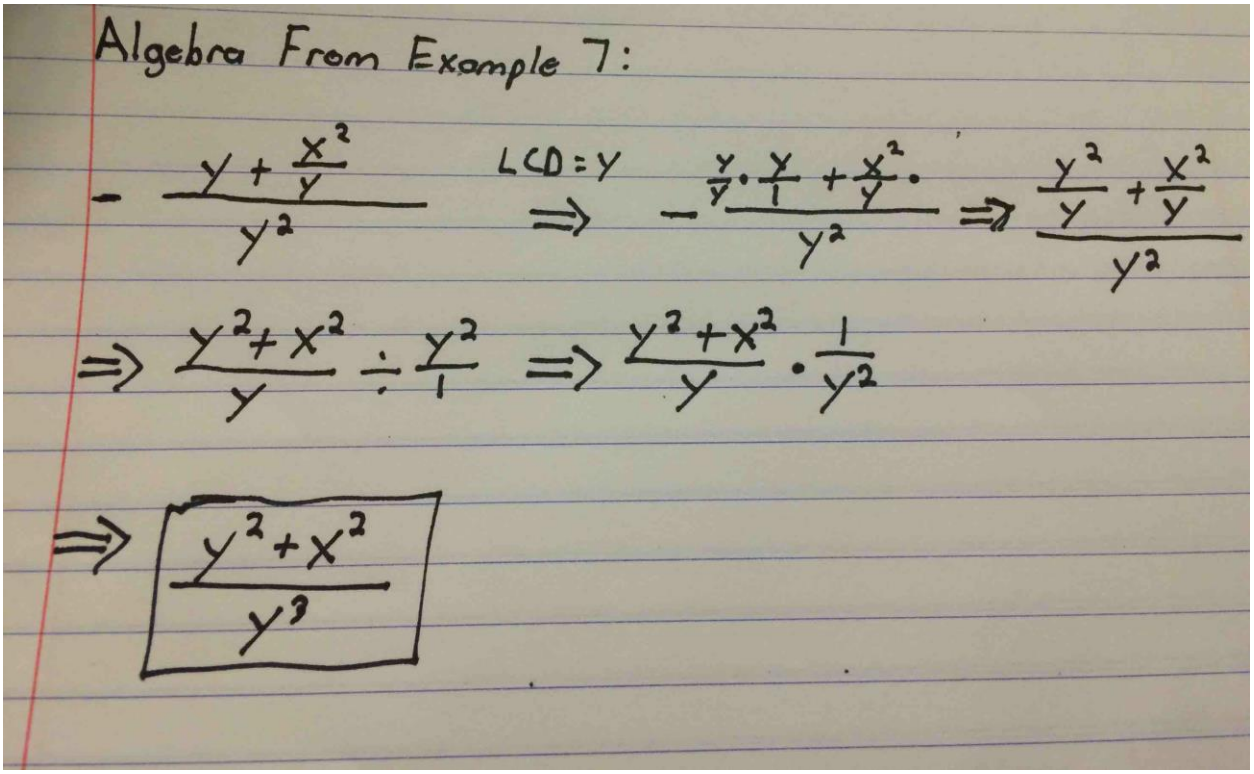
$$12y(x^2+y^2)\frac{dy}{dx}-100x\frac{dy}{dx}=100y-12x(x^2+y^2)$$

$$\frac{dy}{dx}\left[12y(x^2+y^2)-100x\right]=100y-12x(x^2+y^2)$$

$$\frac{dy}{dx}=\frac{100y-12x(x^2+y^2)}{12y(x^2+y^2)-100x}$$

or

$$\frac{25y-3x(x^2+y^2)}{-25x+3y(x^2+y^2)}$$


Algebra From Example 7:
LCD: y