

$$\sin^2 x + \cos^2 x = 1$$

Pythagorean identity

$$\sin^2 x = \frac{1 - \cos 2x}{2}$$

Half-angle identity for $\sin^2 x$

$$\cos^2 x = \frac{1 + \cos 2x}{2}$$

Half-angle identity for $\cos^2 x$

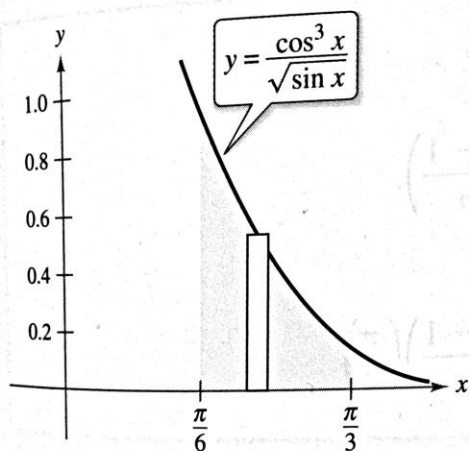
EXAMPLE 1**Power of Sine Is Odd and Positive**

Find $\int \sin^3 x \cos^4 x \, dx$.

EXAMPLE 2**Power of Cosine Is Odd and Positive**

•••▶ See *LarsonCalculus.com* for an interactive version of this type of example.

Evaluate $\int_{\pi/6}^{\pi/3} \frac{\cos^3 x}{\sqrt{\sin x}} dx.$



The area of the region is approximately 0.239.

Figure 8.4

EXAMPLE 3**Power of Cosine Is Even and Nonnegative**

Find $\int \cos^4 x \, dx$.

Wallis's Formulas

1. If n is odd ($n \geq 3$), then

$$\int_0^{\pi/2} \cos^n x \, dx = \left(\frac{2}{3}\right)\left(\frac{4}{5}\right)\left(\frac{6}{7}\right) \cdots \left(\frac{n-1}{n}\right).$$

2. If n is even ($n \geq 2$), then

$$\int_0^{\pi/2} \cos^n x \, dx = \left(\frac{1}{2}\right)\left(\frac{3}{4}\right)\left(\frac{5}{6}\right) \cdots \left(\frac{n-1}{n}\right)\left(\frac{\pi}{2}\right).$$

EXAMPLE 4**Power of Tangent Is Odd and Positive**

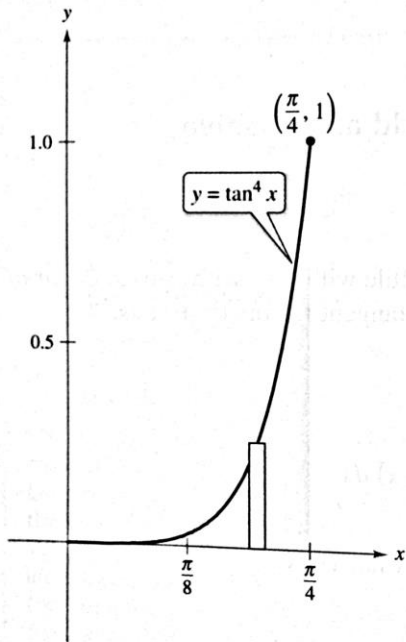
Find $\int \frac{\tan^3 x}{\sqrt{\sec x}} dx.$

EXAMPLE 5**Power of Secant Is Even and Positive**

Find $\int \sec^4 3x \tan^3 3x dx.$

EXAMPLE 6**Power of Tangent Is Even**

Evaluate $\int_0^{\pi/4} \tan^4 x \, dx$.



The area of the region is approximately 0.119.

Figure 8.5

EXAMPLE 7**Converting to Sines and Cosines**

Find $\int \frac{\sec x}{\tan^2 x} dx$.

$$\sin mx \sin nx = \frac{1}{2}(\cos [(m - n)x] - \cos [(m + n)x])$$

$$\sin mx \cos nx = \frac{1}{2}(\sin [(m - n)x] + \sin [(m + n)x])$$

$$\cos mx \cos nx = \frac{1}{2}(\cos [(m - n)x] + \cos [(m + n)x])$$

EXAMPLE 8**Using Product-to-Sum Identities**

Find $\int \sin 5x \cos 4x \, dx$.