THEOREM 8.1 Integration by Parts

If u and v are functions of x and have continuous derivatives, then

$$\int u\,dv = uv - \int v\,du.$$

GUIDELINES FOR INTEGRATION BY PARTS

1. Try letting dv be the most complicated portion of the integrand that fits a basic integration rule. Then u will be the remaining factor(s) of the integrand.
2. Try letting u be the portion of the integrand whose derivative is a function simpler than u. Then dv will be the remaining factor(s) of the integrand.

Note that dv always includes the dx of the original integrand.

EXAMPLE 1 Integration by Parts

Find $\int xe^x\,dx$.

EXAMPLE 2 **Integration by Parts**

Find $\int x^2 \ln x \, dx$.

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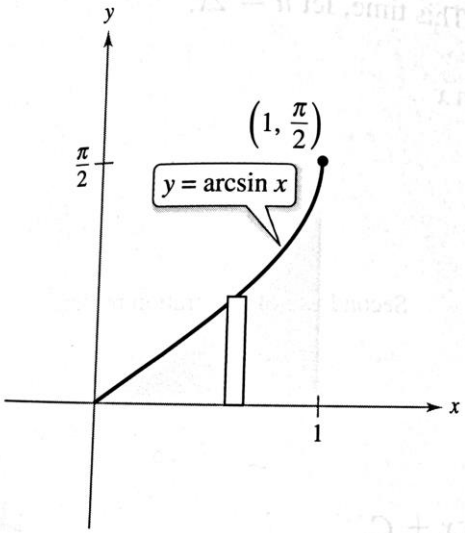

The area of the region is approximately 0.571.

Figure 8.2

EXAMPLE 4 **Repeated Use of Integration by Parts**

Find $\int x^2 \sin x \, dx$.

EXAMPLE 5 **Integration by Parts**

Find $\int \sec^3 x \, dx$.

EXAMPLE 6 **Finding a Centroid**

A machine part is modeled by the region bounded by the graph of $y = \sin x$ and the x-axis, $0 \le x \le \pi/2$, as shown in Figure 8.3. Find the centroid of this region.

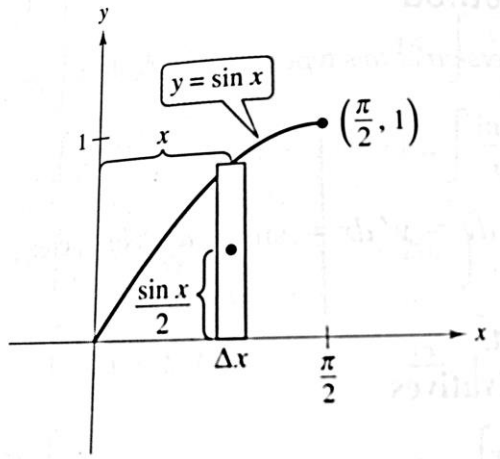

Figure 8.3

SUMMARY: COMMON INTEGRALS USING INTEGRATION BY PARTS

1. For integrals of the form

$$\int x^n e^{ax}\,dx, \quad \int x^n \sin ax\,dx, \quad \text{or} \quad \int x^n \cos ax\,dx$$

let $u = x^n$ and let $dv = e^{ax}\,dx$, $\sin ax\,dx$, or $\cos ax\,dx$.

2. For integrals of the form

$$\int x^n \ln x\,dx, \quad \int x^n \arcsin ax\,dx, \quad \text{or} \quad \int x^n \arctan ax\,dx$$

let $u = \ln x$, $\arcsin ax$, or $\arctan ax$ and let $dv = x^n\,dx$.

3. For integrals of the form

$$\int e^{ax} \sin bx\,dx \quad \text{or} \quad \int e^{ax} \cos bx\,dx$$

let $u = \sin bx$ or $\cos bx$ and let $dv = e^{ax}\,dx$.

EXAMPLE 7 Using the Tabular Method

▶ *See LarsonCalculus.com for an interactive version of this type of example.*

Find $\int x^2 \sin 4x\,dx$.