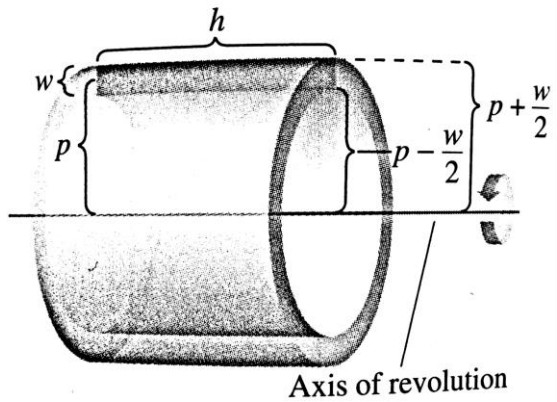
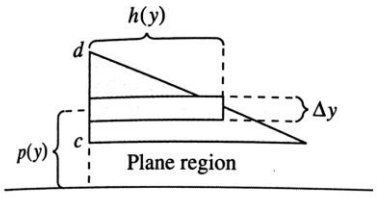


Figure 7.27



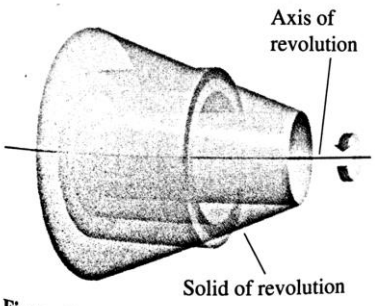


Figure 7.28

THE SHELL METHOD

To find the volume of a solid of revolution with the **shell method,** use one of the formulas below. (See Figure 7.29.)

Horizontal Axis of Revolution	**Vertical Axis of Revolution**
Volume $= V = 2\pi \int_e^d p(y)h(y)\,dy$	Volume $= V = 2\pi \int_a^b p(x)h(x)\,dx$

EXAMPLE 1 **Using the Shell Method to Find Volume**

Find the volume of the solid of revolution formed by revolving the region bounded by

$$y = x - x^3$$

and the x-axis $(0 \le x \le 1)$ about the y-axis.

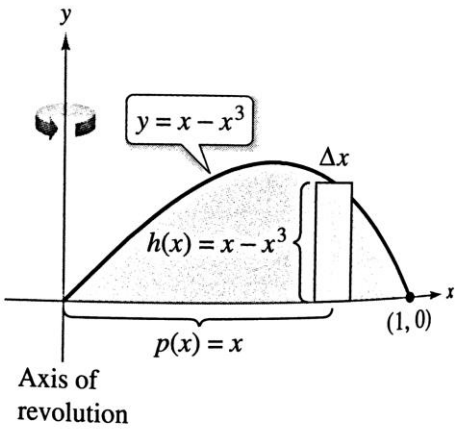


Figure 7.30

EXAMPLE 2 **Using the Shell Method to Find Volume**

Find the volume of the solid of revolution formed by revolving the region bounded by the graph of

$$x = e^{-y^2}$$

and the y-axis $(0 \le y \le 1)$ about the x-axis.

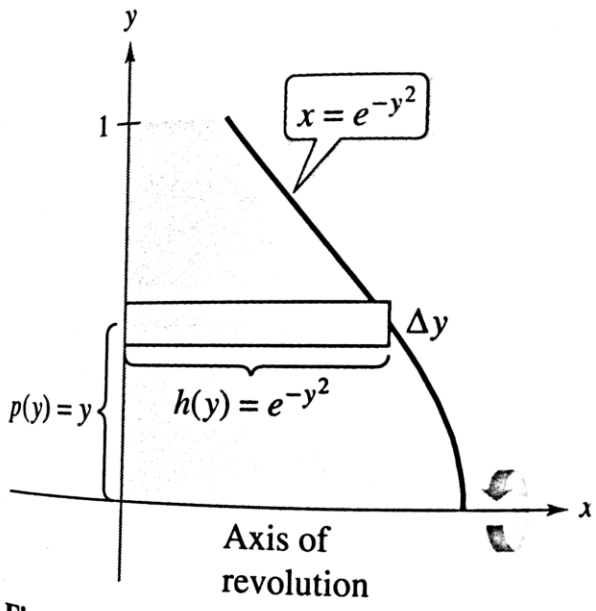
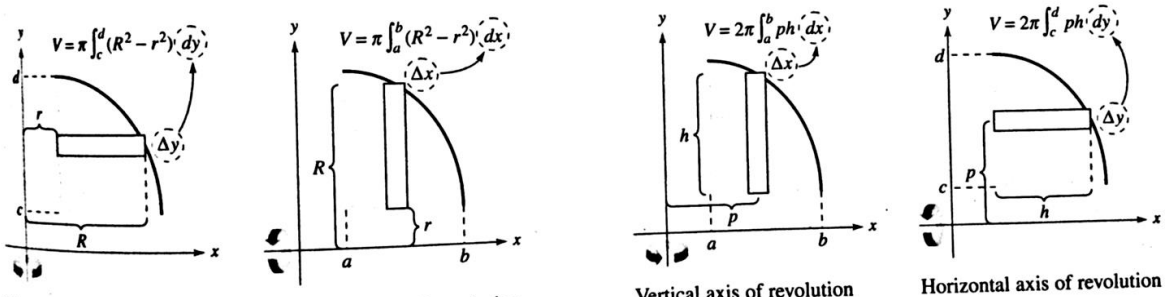


Figure 7.31



Disk method: Representative rectangle is perpendicular to the axis of revolution.

Shell method: Representative rectangle is parallel to the axis of revolution.

Figure 7.32

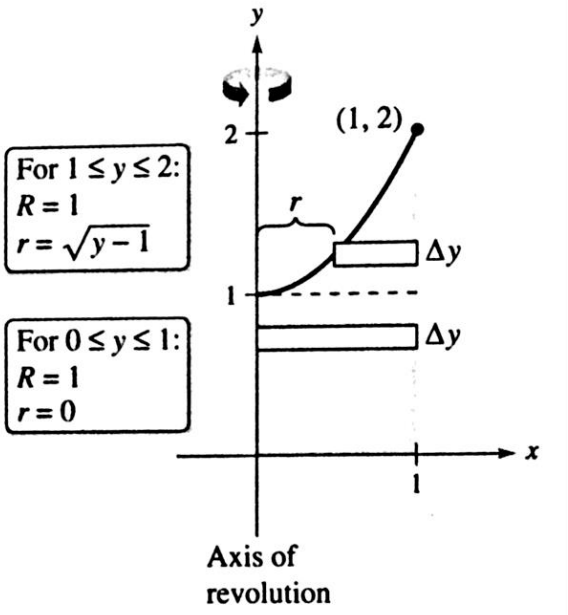
EXAMPLE 3 Shell Method Preferable

See LarsonCalculus.com for an interactive version of this type of example.

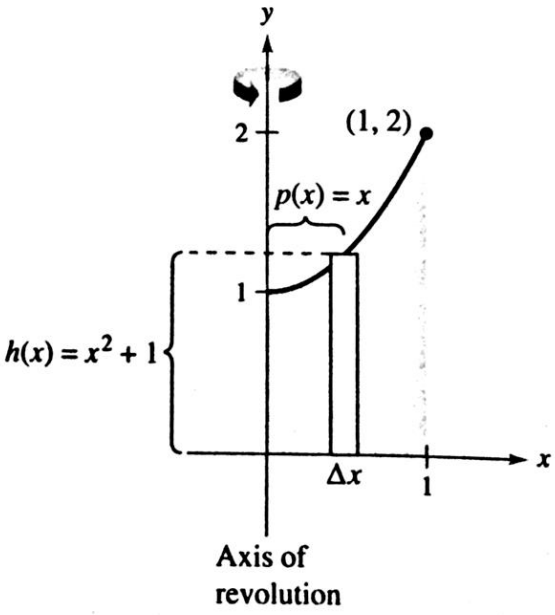
Find the volume of the solid formed by revolving the region bounded by the graphs of

$$y = x^2 + 1, \quad y = 0, \quad x = 0, \quad \text{and} \quad x = 1$$

about the y-axis.



(a) Disk method



(b) Shell method

Figure 7.33

EXAMPLE 4 **Volume of a Pontoon**

A pontoon is to be made in the shape shown in Figure 7.34. The pontoon is designed by rotating the graph of

$$y = 1 - \frac{x^2}{16}, \quad -4 \le x \le 4$$

about the x-axis, where x and y are measured in feet. Find the volume of the pontoon.

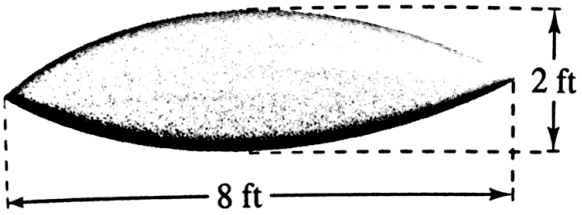
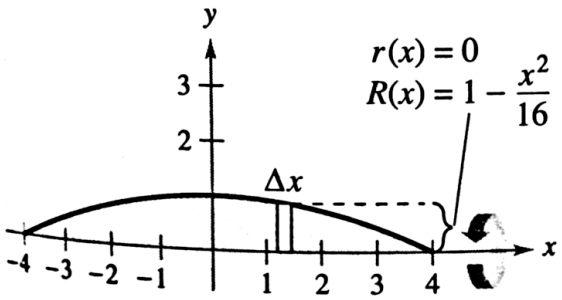


Figure 7.34



Disk method

Figure 7.35

EXAMPLE 5 **Shell Method Necessary**

Find the volume of the solid formed by revolving the region bounded by the graphs of $y = x^3 + x + 1$, $y = 1$, and $x = 1$ about the line $x = 2$, as shown in Figure 7.36.

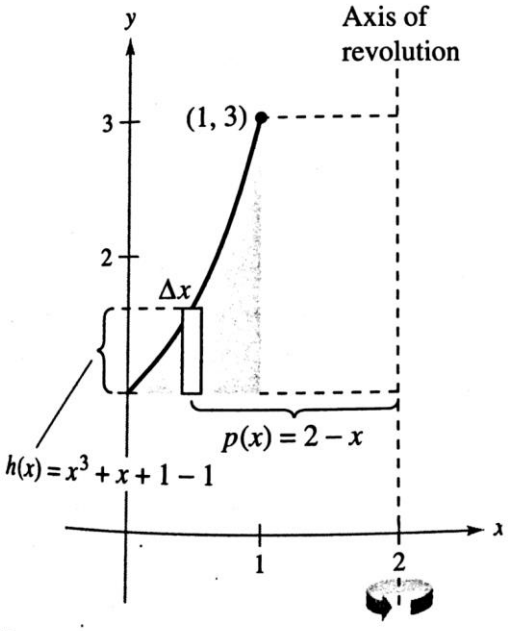


Figure 7.36