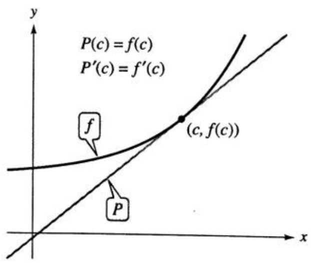
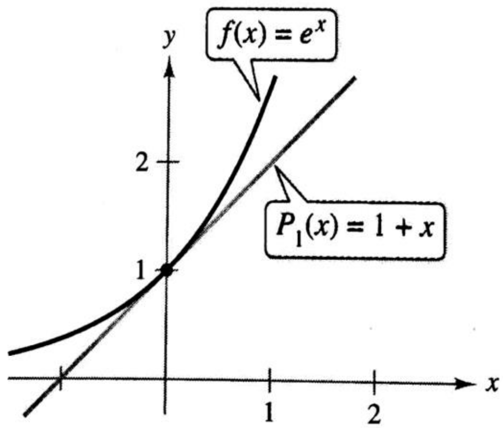


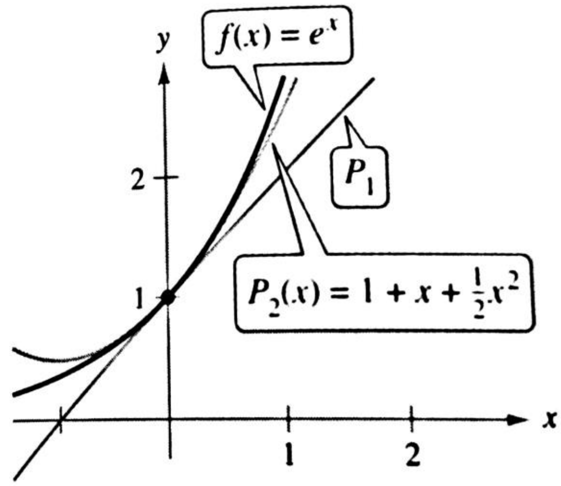
Near $(c, f(c))$, the graph of P can be used to approximate the graph of f.
Figure 9.10

EXAMPLE 1 First-Degree Polynomial Approximation of $f(x) = e^x$

For the function $f(x) = e^x$, find a first-degree polynomial function $P_1(x) = a_0 + a_1 x$ whose value and slope agree with the value and slope of f at $x = 0$.



P_1 is the first-degree polynomial approximation of $f(x) = e^x$.
Figure 9.11



P_2 is the second-degree polynomial approximation of $f(x) = e^x$.
Figure 9.12.

EXAMPLE 2 Third-Degree Polynomial Approximation of $f(x) = e^x$

Construct a table comparing the values of the polynomial

$$P_3(x) = 1 + x + \frac{1}{2}x^2 + \frac{1}{3!}x^3$$ 3rd-degree approximation

with $f(x) = e^x$ for several values of x near 0.

Definitions of *n*th Taylor Polynomial and *n*th Maclaurin Polynomial

If f has n derivatives at c, then the polynomial

$$P_n(x) = f(c) + f'(c)(x - c) + \frac{f''(c)}{2!}(x - c)^2 + \cdots + \frac{f^{(n)}(c)}{n!}(x - c)^n$$

is called the ***n*th Taylor polynomial for *f* at *c*.** If $c = 0$, then

$$P_n(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \cdots + \frac{f^{(n)}(0)}{n!}x^n$$

is also called the ***n*th Maclaurin polynomial for *f*.**

EXAMPLE 3 **A Maclaurin Polynomial for $f(x) = e^x$**

Find the nth Maclaurin polynomial for

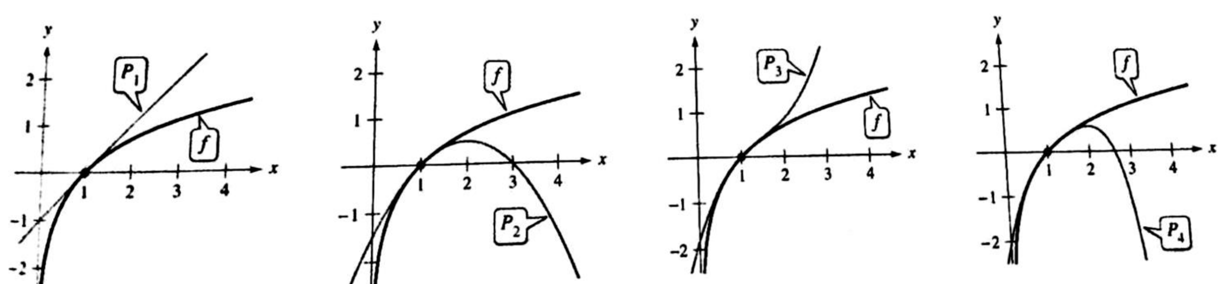
$f(x) = e^x$.

EXAMPLE 4 Finding Taylor Polynomials for ln *x*

Find the Taylor polynomials P_0, P_1, P_2, P_3, and P_4 for

$$f(x) = \ln x$$

centered at $c = 1$.

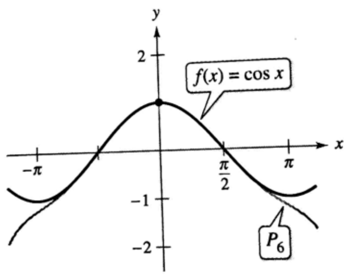


As n increases, the graph of P_n becomes a better and better approximation of the graph of $f(x) = \ln x$ near $x = 1$.

Figure 9.14

EXAMPLE 5 Finding Maclaurin Polynomials for cos *x*

Find the Maclaurin polynomials P_0, P_2, P_4, and P_6 for $f(x) = \cos x$. Use $P_6(x)$ to approximate the value of $\cos(0.1)$.

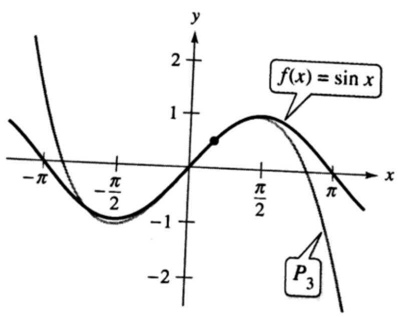


Near (0, 1), the graph of P_6 can be used to approximate the graph of $f(x) = \cos x$.
Figure 9.15

EXAMPLE 6 Finding a Taylor Polynomial for sin *x*

See LarsonCalculus.com for an interactive version of this type of example.

Find the third Taylor polynomial for $f(x) = \sin x$, expanded about $c = \pi/6$.



Near $(\pi/6, 1/2)$, the graph of P_3 can be used to approximate the graph of $f(x) = \sin x$.

Figure 9.16

EXAMPLE 7 **Approximation Using Maclaurin Polynomials**

Use a fourth Maclaurin polynomial to approximate the value of $\ln(1.1)$.

$$f(x) = P_n(x) + R_n(x)$$

Exact value — Approximate value — Remainder

). The absolute value of $R_n(x)$ is called the **erro**
That is,

$$\text{Error} = |R_n(x)| = |f(x) - P_n(x)|.$$

THEOREM 9.19 Taylor's Theorem

If a function f is differentiable through order $n + 1$ in an interval I containing c, then, for each x in I, there exists z between x and c such that

$$f(x) = f(c) + f'(c)(x - c) + \frac{f''(c)}{2!}(x - c)^2 + \cdots + \frac{f^{(n)}(c)}{n!}(x - c)^n + R_n(x)$$

where

$$R_n(x) = \frac{f^{(n+1)}(z)}{(n + 1)!}(x - c)^{n+1}.$$

A proof of this theorem is given in Appendix A.

See LarsonCalculus.com for Bruce Edwards's video of this proof.

EXAMPLE 8 Determining the Accuracy of an Approximation

The third Maclaurin polynomial for $\sin x$ is

$$P_3(x) = x - \frac{x^3}{3!}.$$

Use Taylor's Theorem to approximate $\sin(0.1)$ by $P_3(0.1)$ and determine the accuracy of the approximation.

EXAMPLE 9 Approximating a Value to a Desired Accuracy

Determine the degree of the Taylor polynomial $P_n(x)$ expanded about $c = 1$ that should be used to approximate $\ln(1.2)$ so that the error is less than 0.001.