Newton's Method for Approximating the Zeros of a Function

Let $f(c) = 0$, where f is differentiable on an open interval containing c. Then, to approximate c, use these steps.

1. Make an initial estimate x_1 that is close to c. (A graph is helpful.)
2. Determine a new approximation

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}.$$

3. When $|x_n - x_{n+1}|$ is within the desired accuracy, let x_{n+1} serve as the final approximation. Otherwise, return to Step 2 and calculate a new approximation.

Each successive application of this procedure is called an **iteration.**

EXAMPLE 1 Using Newton's Method

Calculate three iterations of Newton's Method to approximate a zero of $f(x) = x^2 - 2$. Use $x_1 = 1$ as the initial guess.

EXAMPLE 2 Using Newton's Method

See LarsonCalculus.com for an interactive version of this type of example.

Use Newton's Method to approximate the zeros of

$$f(x) = 2x^3 + x^2 - x + 1.$$

Continue the iterations until two successive approximations differ by less than 0.0001

Using Newton's Method **In Exercises 1–4, complete two iterations of Newton's Method to approximate a zero of the function using the given initial guess.**

1. $f(x) = x^2 - 5, \quad x_1 = 2.2$

2. $f(x) = x^3 - 3, \quad x_1 = 1.4$

Using Newton's Method **In Exercises 5–14, approximate the zero(s) of the function. Use Newton's Method and continue the process until two successive approximations differ by less than 0.001. Then find the zero(s) using a graphing utility and compare the results.**

5. $f(x) = x^3 + 4$

6. $f(x) = 2 - x^3$

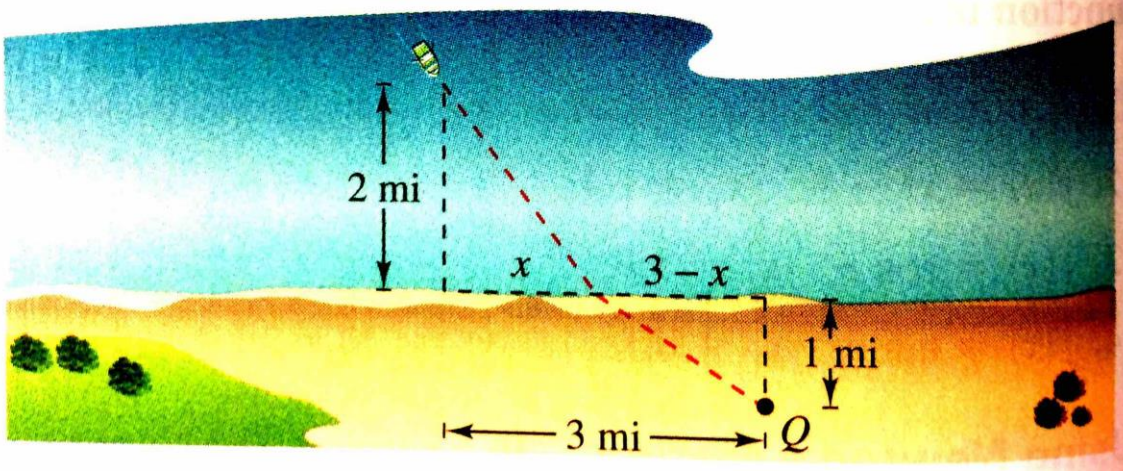
7. $f(x) = x^3 + x - 1$

8. $f(x) = x^5 + x - 1$

EXAMPLE 3 **An Example in Which Newton's Method Fails**

The function $f(x) = x^{1/3}$ is not differentiable at $x = 0$. Show that Newton's Method fails to converge using $x_1 = 0.1$.

Minimum Time You are in a boat 2 miles from the nearest point on the coast (see figure). You are to go to a point Q that is 3 miles down the coast and 1 mile inland. You can row at 3 miles per hour and walk at 4 miles per hour. Toward what point on the coast should you row in order to reach Q in the least time?



Crime The total number of arrests T (in thousands) for all males ages 15 to 24 in 2010 is approximated by the model

$$T = 0.2988x^4 - 22.625x^3 + 628.49x^2 - 7565.9x + 33{,}478$$

for $15 \le x \le 24$, where x is the age in years (see figure). Approximate the two ages that had total arrests of 300 thousand. *(Source: U.S. Department of Justice)*