THEOREM 1.11 Properties of Continuity

If b is a real number and f and g are continuous at $x = c$, then the functions listed below are also continuous at c.

1. Scalar multiple: bf

2. Sum or difference: $f \pm g$

3. Product: fg

4. Quotient: $\frac{f}{g}$, $g(c) \neq 0$

EXAMPLE 7 Testing for Continuity

Describe the interval(s) on which each function is continuous.

a. $f(x) = \tan x$

b. $g(x) = \begin{cases} \sin \frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$

c. $h(x) = \begin{cases} x \sin \frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$

THEOREM 1.13 Intermediate Value Theorem

If f is continuous on the closed interval $[a, b]$, $f(a) \neq f(b)$, and k is any number between $f(a)$ and $f(b)$, then there is at least one number c in $[a, b]$ such that

$$f(c) = k.$$

EXAMPLE 8 An Application of the Intermediate Value Theorem

Use the Intermediate Value Theorem to show that the polynomial function

$$f(x) = x^3 + 2x - 1$$

has a zero in the interval $[0, 1]$.