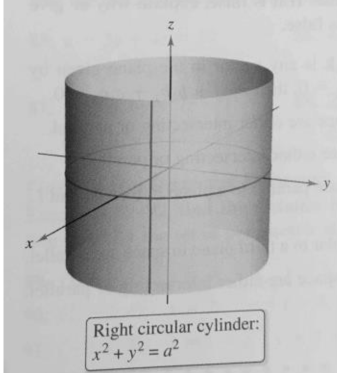
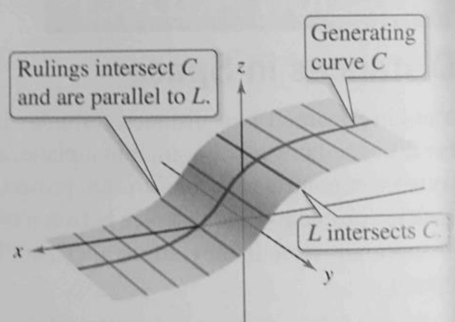


Rulings are parallel to z-axis
Figure 11.56

Definition of a Cylinder

Let C be a curve in a plane and let L be a line not in a parallel plane. The set of all lines parallel to L and intersecting C is a **cylinder**. The curve C is the **generating curve** (or **directrix**) of the cylinder, and the parallel lines are **rulings.**



Right cylinder: A cylinder whose rulings are perpendicular to the coordinate plane containing C
Figure 11.57

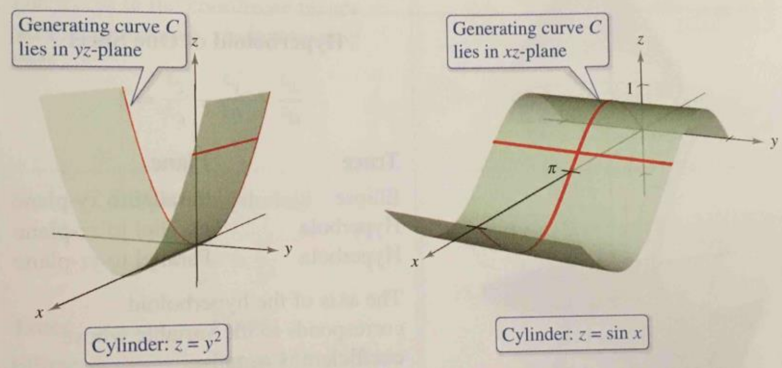
Equations of Cylinders

The equation of a cylinder whose rulings are parallel to one of the coordinate axes contains only the variables corresponding to the other two axes.

EXAMPLE 1 Sketching a Cylinder

Sketch the surface represented by each equation.

a. $z = y^2$ **b.** $z = \sin x,\quad 0 \le x \le 2\pi$



(a) Rulings are parallel to x-axis. (b) Rulings are parallel to y-axis.

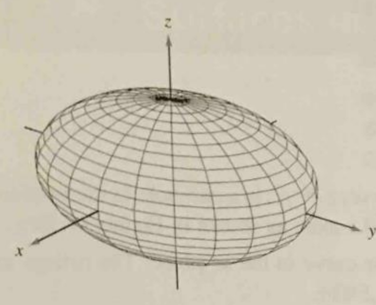
Figure 11.58

Quadric Surface

The equation of a **quadric surface** in space is a second-degree equation in three variables. The **general form** of the equation is

$$Ax^2 + By^2 + Cz^2 + Dxy + Exz + Fyz + Gx + Hy + Iz + J = 0.$$

There are six basic types of quadric surfaces: **ellipsoid, hyperboloid of one sheet, hyperboloid of two sheets, elliptic cone, elliptic paraboloid, and hyperbolic paraboloid.**

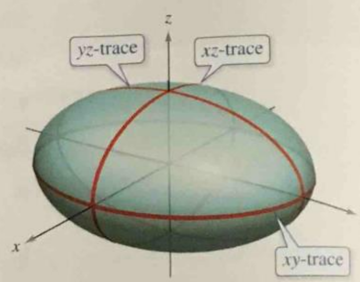


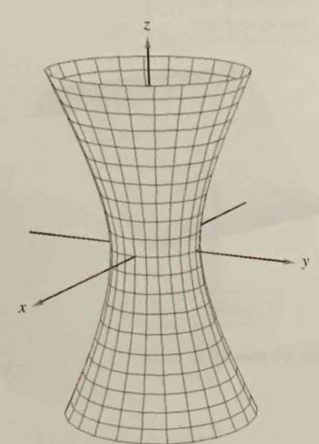
Ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

Trace	**Plane**
Ellipse	Parallel to xy-plane
Ellipse	Parallel to xz-plane
Ellipse	Parallel to yz-plane

The surface is a sphere when $a = b = c \neq 0$.



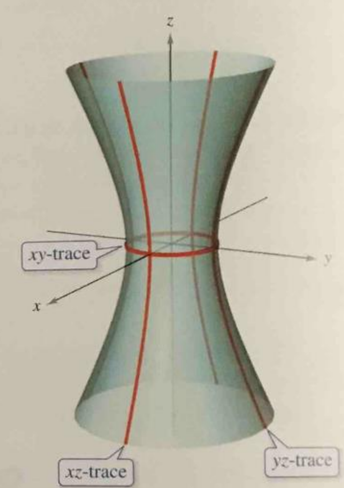


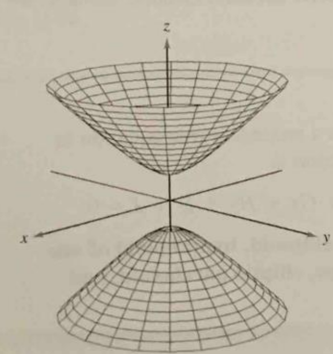
Hyperboloid of One Sheet

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$$

Trace	**Plane**
Ellipse	Parallel to xy-plane
Hyperbola	Parallel to xz-plane
Hyperbola	Parallel to yz-plane

The axis of the hyperboloid corresponds to the variable whose coefficient is negative.



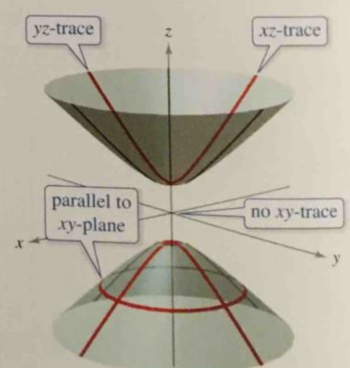


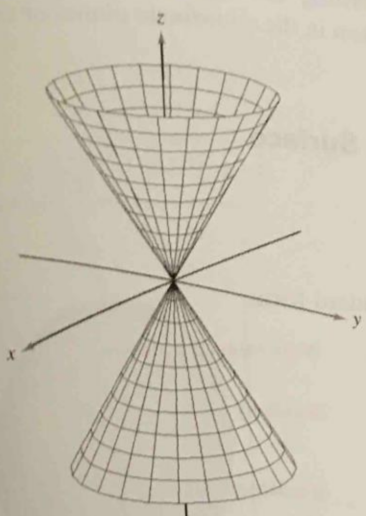
Hyperboloid of Two Sheets

$$\frac{z^2}{c^2} - \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Trace	**Plane**
Ellipse	Parallel to xy-plane
Hyperbola	Parallel to xz-plane
Hyperbola	Parallel to yz-plane

The axis of the hyperboloid corresponds to the variable whose coefficient is positive. There is no trace in the coordinate plane perpendicular to this axis.



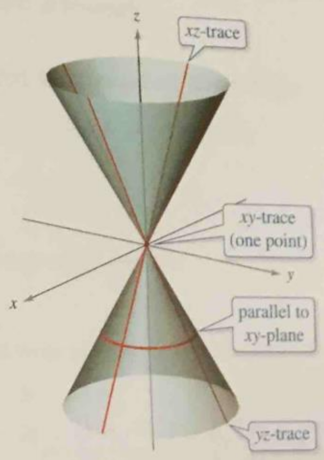


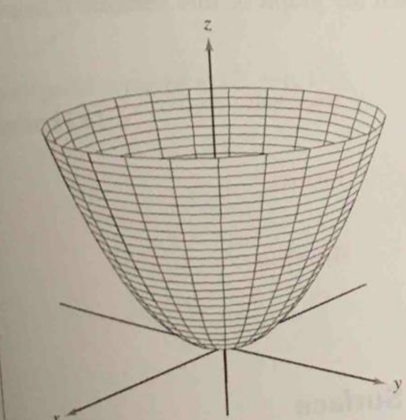
Elliptic Cone

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 0$$

Trace	Plane
Ellipse	Parallel to xy-plane
Hyperbola	Parallel to xz-plane
Hyperbola	Parallel to yz-plane

The axis of the cone corresponds to the variable whose coefficient is negative. The traces in the coordinate planes parallel to this axis are intersecting lines.



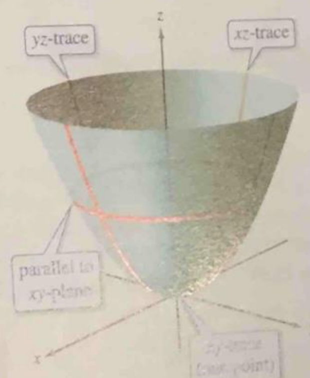


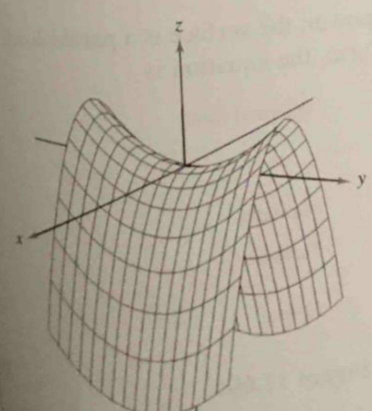
Elliptic Paraboloid

$$z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$$

Trace	Plane
Ellipse	Parallel to xy-plane
Parabola	Parallel to xz-plane
Parabola	Parallel to yz-plane

The axis of the paraboloid corresponds to the variable raised to the first power.



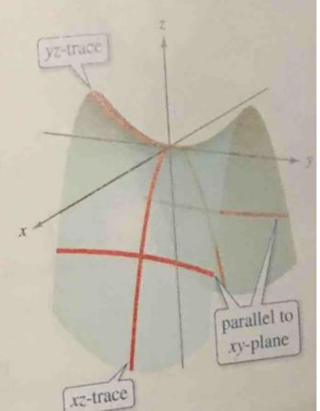


Hyperbolic Paraboloid

$$z = \frac{y^2}{b^2} - \frac{x^2}{a^2}$$

Trace	Plane
Hyperbola	Parallel to xy-plane
Parabola	Parallel to xz-plane
Parabola	Parallel to yz-plane

The axis of the paraboloid corresponds to the variable raised to the first power.



EXAMPLE 2 **Sketching a Quadric Surface**

Classify and sketch the surface

$$4x^2 - 3y^2 + 12z^2 + 12 = 0.$$

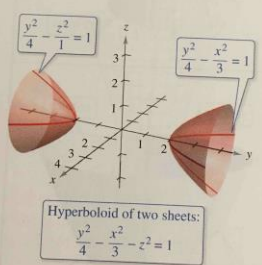


Figure 11.59

EXAMPLE 3 **Sketching a Quadric Surface**

Classify and sketch the surface

$$x - y^2 - 4z^2 = 0.$$

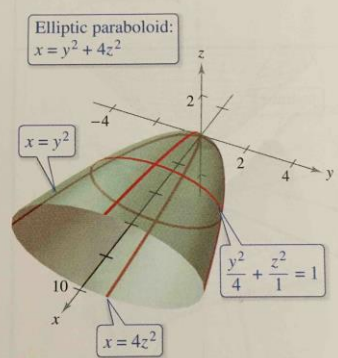


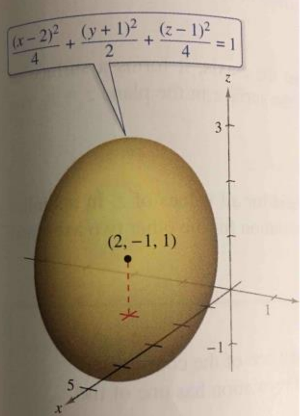
Figure 11.60

EXAMPLE 4 **A Quadric Surface Not Centered at the Origin**

▷ *See LarsonCalculus.com for an interactive version of this type of example.*

Classify and sketch the surface

$$x^2 + 2y^2 + z^2 - 4x + 4y - 2z + 3 = 0.$$



An ellipsoid centered at (2, −1, 1)
Figure 11.61

Surface of Revolution

If the graph of a radius function r is revolved about one of the coordinate axes, then the equation of the resulting surface of revolution has one of the forms listed below.

1. Revolved about the x-axis: $y^2 + z^2 = [r(x)]^2$
2. Revolved about the y-axis: $x^2 + z^2 = [r(y)]^2$
3. Revolved about the z-axis: $x^2 + y^2 = [r(z)]^2$

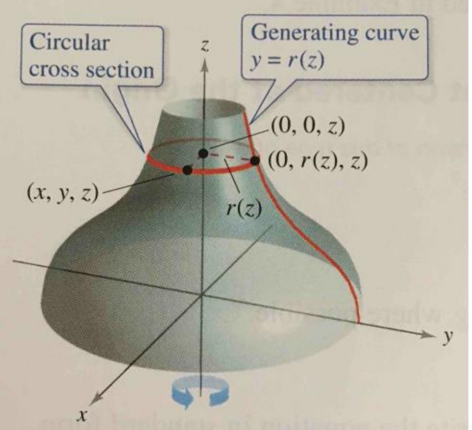


Figure 11.62

EXAMPLE 5 **Finding an Equation for a Surface of Revolution**

Find an equation for the surface of revolution formed by revolving (a) the graph of $y = 1/z$ about the z-axis and (b) the graph of $9x^2 = y^3$ about the y-axis.

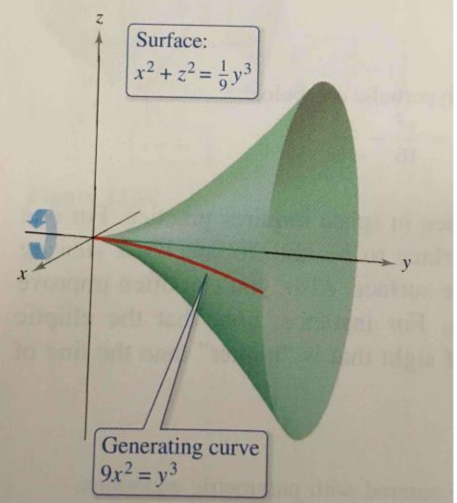


Figure 11.63

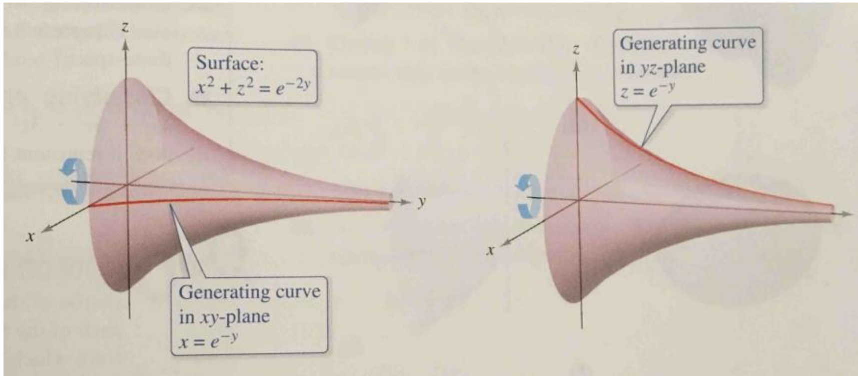


Figure 11.64

EXAMPLE 6 Finding a Generating Curve

Find a generating curve and the axis of revolution for the surface

$$x^2 + 3y^2 + z^2 = 9.$$

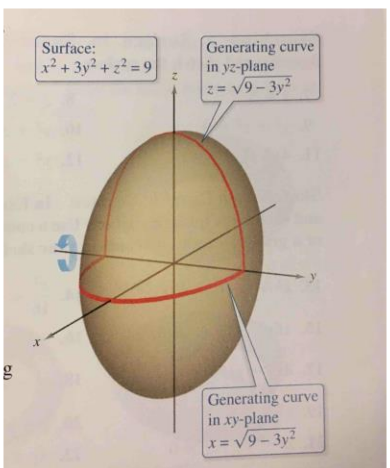


Figure 11.65

The Cylindrical Coordinate System

In a **cylindrical coordinate system,** a point P in space is represented by an ordered triple (r, θ, z).

1. (r, θ) is a polar representation of the projection of P in the xy-plane.
2. z is the directed distance from (r, θ) to P.

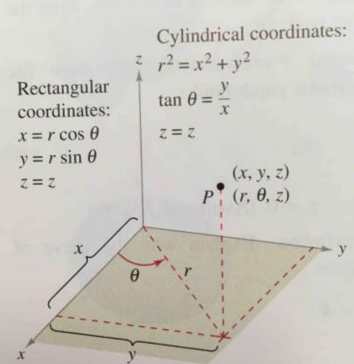


Figure 11.66

Cylindrical to rectangular:

$$x = r\cos\theta, \qquad y = r\sin\theta, \qquad z = z$$

Rectangular to cylindrical:

$$r^2 = x^2 + y^2, \qquad \tan\theta = \frac{y}{x}, \qquad z = z$$

EXAMPLE 1 **Cylindrical-to-Rectangular Conversion**

Convert the point $(r, \theta, z) = (4, 5\pi/6, 3)$ to rectangular coordinates.

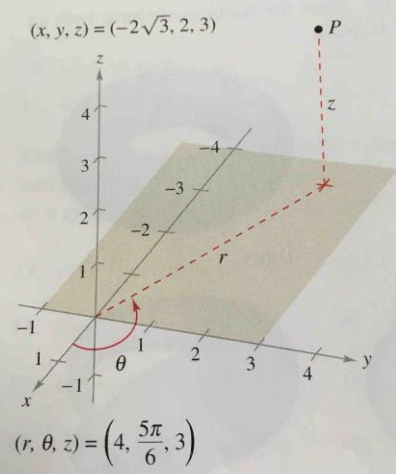


Figure 11.67

EXAMPLE 2 **Rectangular-to-Cylindrical Conversion**

Convert the point

$$(x, y, z) = \left(1, \sqrt{3}, 2\right)$$

to cylindrical coordinates.

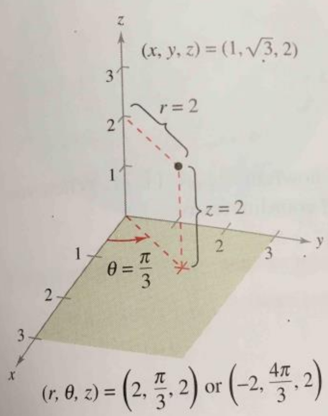


Figure 11.68

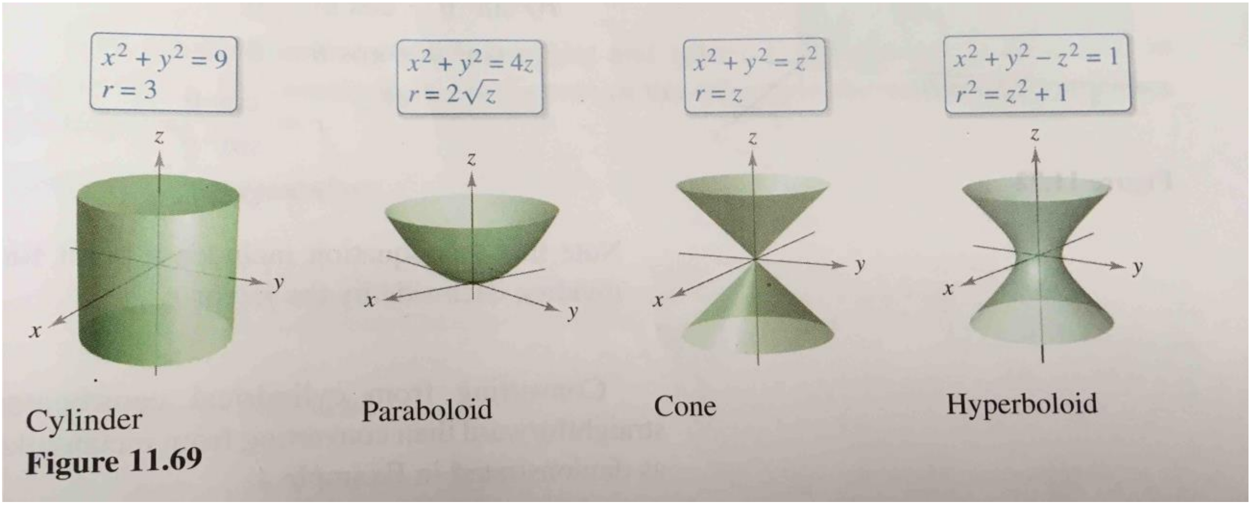


Figure 11.69

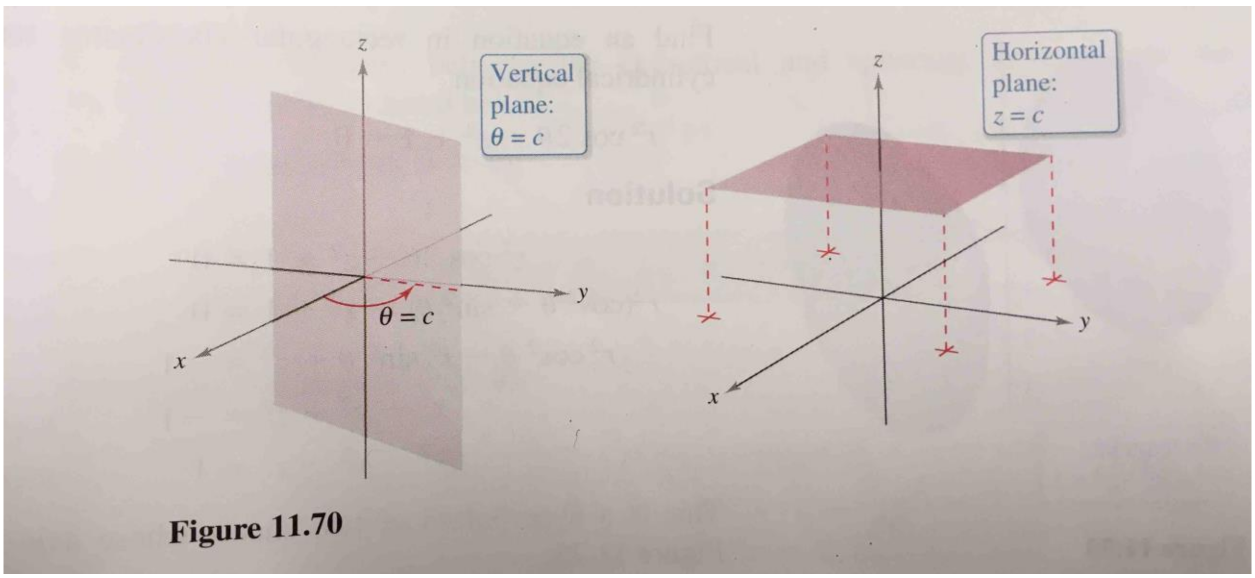


Figure 11.70

EXAMPLE 3 Rectangular-to-Cylindrical Conversion

Find an equation in cylindrical coordinates for the surface represented by each rectangular equation.

a. $x^2 + y^2 = 4z^2$

b. $y^2 = x$

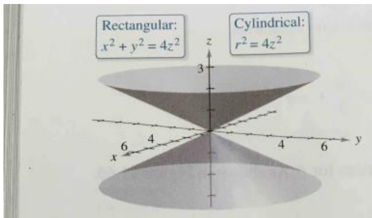


Figure 11.71

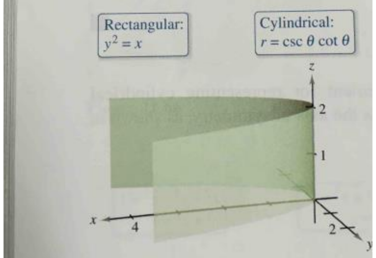


Figure 11.72

EXAMPLE 4 **Cylindrical-to-Rectangular Conversion**

Find an equation in rectangular coordinates for the surface represented by the cylindrical equation

$$r^2 \cos 2\theta + z^2 + 1 = 0.$$

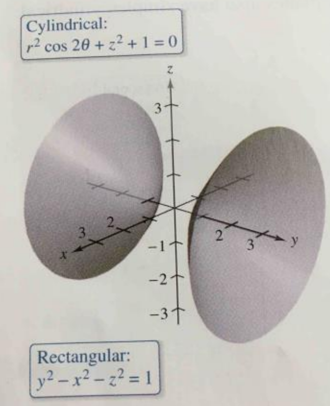


Figure 11.73

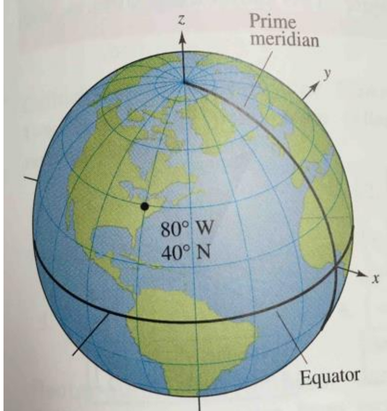


Figure 11.74

$(4000, -80°, 50°)$.

Radius — 80° clockwise from prime meridian — 50° down from North Pole

The Spherical Coordinate System

In a **spherical coordinate system,** a point P in space is represented by an ordered triple (ρ, θ, ϕ), where ρ is the lowercase Greek letter *rho* and ϕ is the lowercase Greek letter *phi.*

1. ρ is the distance between P and the origin, $\rho \geq 0$.

2. θ is the same angle used in cylindrical coordinates for $r \geq 0$.

3. ϕ is the angle *between* the positive z-axis and the line segment $\overrightarrow{OP}$, $0 \leq \phi \leq \pi$.

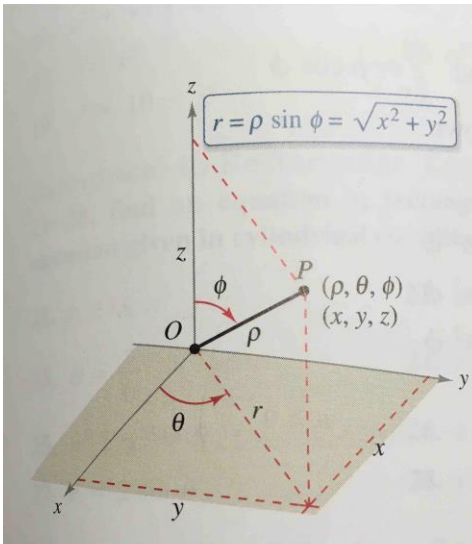
Note that the first and third coordinates, ρ and ϕ, are nonnegative.

Spherical to rectangular:

$$x = \rho \sin \phi \cos \theta, \quad y = \rho \sin \phi \sin \theta, \quad z = \rho \cos \phi$$

Rectangular to spherical:

$$\rho^2 = x^2 + y^2 + z^2, \quad \tan \theta = \frac{y}{x}, \quad \phi = \arccos\left(\frac{z}{\sqrt{x^2 + y^2 + z^2}}\right)$$



Spherical coordinates
Figure 11.75

Spherical to cylindrical **($r \geq 0$):**

$$r^2 = \rho^2 \sin^2\phi, \quad \theta = \theta, \quad z = \rho\cos\phi$$

Cylindrical to spherical **($r \geq 0$):**

$$\rho = \sqrt{r^2 + z^2}, \quad \theta = \theta, \quad \phi = \arccos\left(\frac{z}{\sqrt{r^2 + z^2}}\right)$$

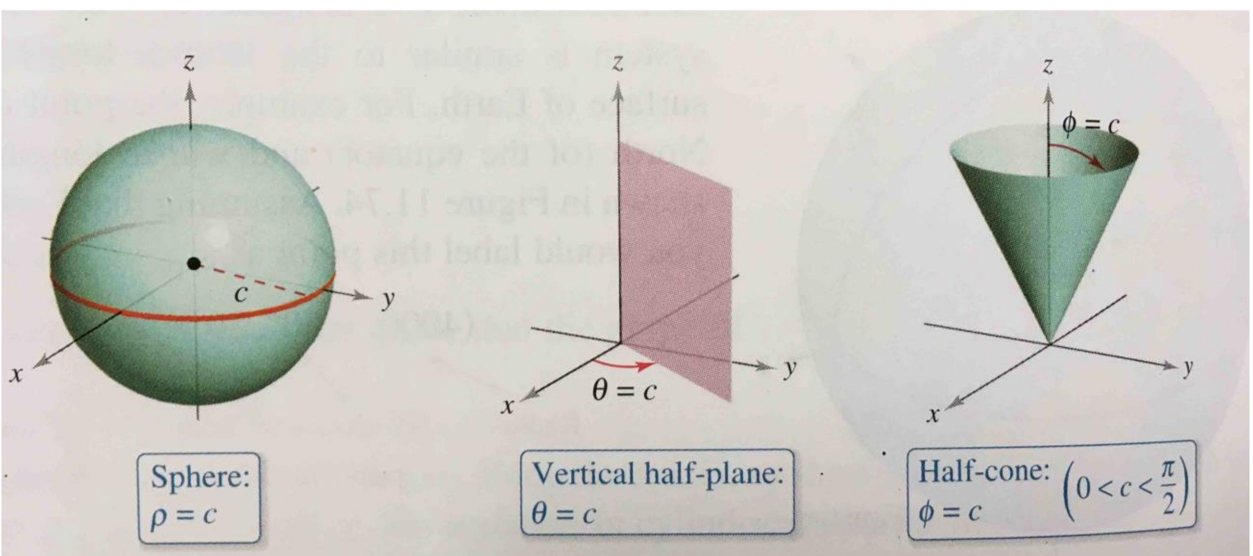


Figure 11.76

EXAMPLE 5 Rectangular-to-Spherical Conversion

See LarsonCalculus.com for an interactive version of this type of example.

Find an equation in spherical coordinates for the surface represented by each rectangular equation.

a. Cone: $x^2 + y^2 = z^2$ **b.** Sphere: $x^2 + y^2 + z^2 - 4z = 0$

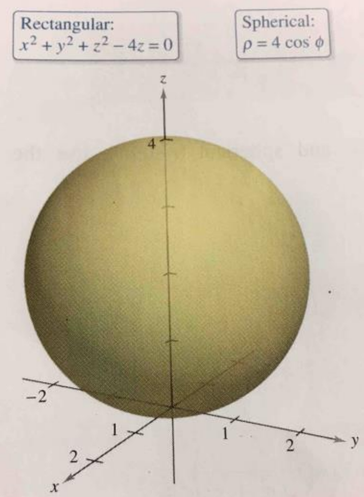


Figure 11.77

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Definition of Vector-Valued Function

A function of the form

$$\mathbf{r}(t) = f(t)\mathbf{i} + g(t)\mathbf{j} \qquad \text{Plane}$$

or

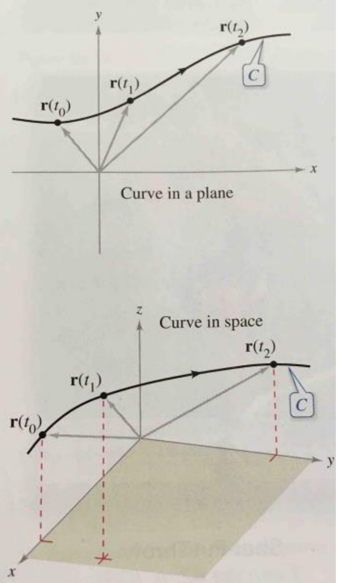
$$\mathbf{r}(t) = f(t)\mathbf{i} + g(t)\mathbf{j} + h(t)\mathbf{k} \qquad \text{Space}$$

is a **vector-valued function,** where the **component functions** f, g, and h are real-valued functions of the parameter t. Vector-valued functions are sometimes denoted as

$$\mathbf{r}(t) = \langle f(t), g(t) \rangle \qquad \text{Plane}$$

or

$$\mathbf{r}(t) = \langle f(t), g(t), h(t) \rangle. \qquad \text{Space}$$

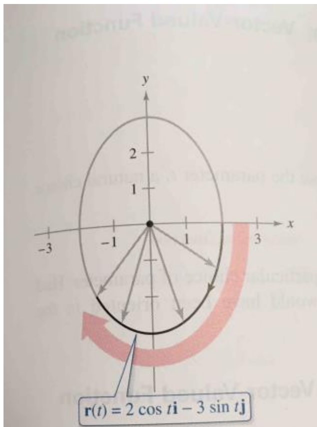


Curve C is traced out by the terminal point of position vector $\mathbf{r}(t)$.
Figure 12.1

EXAMPLE 1 Sketching a Plane Curve

Sketch the plane curve represented by the vector-valued function

$$\mathbf{r}(t) = 2\cos t\mathbf{i} - 3\sin t\mathbf{j}, \quad 0 \le t \le 2\pi. \qquad \text{Vector-valued function}$$



The ellipse is traced clockwise as t increases from 0 to 2π.

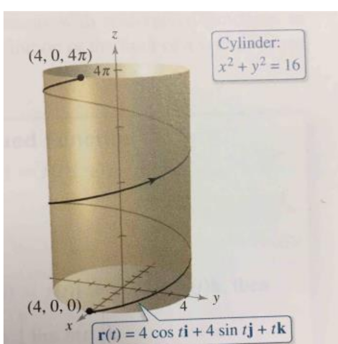
Figure 12.2

EXAMPLE 2 Sketching a Space Curve

••••▷ *See LarsonCalculus.com for an interactive version of this type of example.*

Sketch the space curve represented by the vector-valued function

$$\mathbf{r}(t) = 4\cos t\mathbf{i} + 4\sin t\mathbf{j} + t\mathbf{k}, \quad 0 \le t \le 4\pi. \qquad \text{Vector-valued function}$$



As t increases from 0 to 4π, two spirals on the helix are traced out.

Figure 12.3

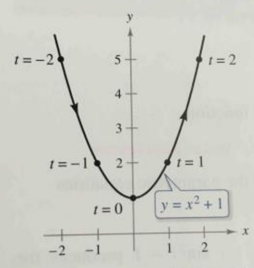
In 1953, Francis Crick and James D. Watson discovered the double helix structure of DNA.

EXAMPLE 3 Representing a Graph: Vector-Valued Function

Represent the parabola

$$y = x^2 + 1$$

by a vector-valued function.



There are many ways to parametrize this graph. One way is to let $x = t$.

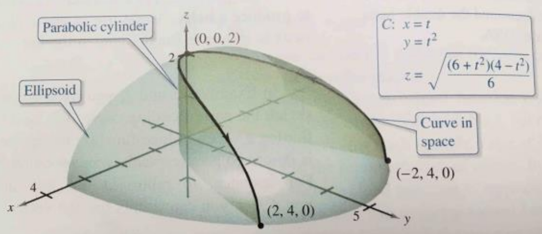
Figure 12.4

EXAMPLE 4 **Representing a Graph: Vector-Valued Function**

Sketch the space curve C represented by the intersection of the semiellipsoid

$$\frac{x^2}{12} + \frac{y^2}{24} + \frac{z^2}{4} = 1, \quad z \geq 0$$

and the parabolic cylinder $y = x^2$. Then find a vector-valued function to represent the graph.



The curve C is the intersection of the semiellipsoid and the parabolic cylinder.
Figure 12.5

Definition of Continuity of a Vector-Valued Function

A vector-valued function $\mathbf{r}$ is **continuous at the point** given by $t = a$ when the limit of $\mathbf{r}(t)$ exists as $t \to a$ and

$$\lim_{t \to a} \mathbf{r}(t) = \mathbf{r}(a).$$

A vector-valued function $\mathbf{r}$ is **continuous on an interval** I when it is continuous at every point in the interval.

EXAMPLE 5 **Continuity of a Vector-Valued Function**

Discuss the continuity of the vector-valued function

$a = 4$

$$\mathbf{r}(t) = t\mathbf{i} + a\mathbf{j} + (a^2 - t^2)\mathbf{k} \qquad a \text{ is a constant.}$$

at $t = 0$.

EXAMPLE 6 **Continuity of a Vector-Valued Function**

Determine the interval(s) on which the vector-valued function

$$\mathbf{r}(t) = t\mathbf{i} + \sqrt{t+1}\,\mathbf{j} + (t^2 + 1)\mathbf{k}$$

is continuous.