

EXAMPLE 1**A Comparison of Three Similar Integrals**

•••▶ See *LarsonCalculus.com* for an interactive version of this type of example.

Find each integral.

a. $\int \frac{4}{x^2 + 9} dx$ b. $\int \frac{4x}{x^2 + 9} dx$ c. $\int \frac{4x^2}{x^2 + 9} dx$

Solution

EXAMPLE 2**Using Two Basic Rules to Solve a Single Integral**

Evaluate $\int_0^1 \frac{x+3}{\sqrt{4-x^2}} dx$.

EXAMPLE 3**A Substitution Involving $a^2 - u^2$**

Find $\int \frac{x^2}{\sqrt{16-x^6}} dx$.

EXAMPLE 4**A Disguised Form of the Log Rule**

Find $\int \frac{1}{1 + e^x} dx$.

EXAMPLE 5**A Disguised Form of the Power Rule**

Find $\int (\cot x)[\ln(\sin x)] dx$.

EXAMPLE 6**Using Trigonometric Identities**

Find $\int \tan^2 2x \, dx$.

PROCEDURES FOR FITTING INTEGRANDS TO BASIC INTEGRATION RULES**Technique**

Expand (numerator).

Separate numerator.

Complete the square.

Divide improper rational function.

Add and subtract terms in numerator.

Use trigonometric identities.

Multiply and divide by Pythagorean conjugate.

Example

$$(1 + e^x)^2 = 1 + 2e^x + e^{2x}$$

$$\frac{1+x}{x^2+1} = \frac{1}{x^2+1} + \frac{x}{x^2+1}$$

$$\frac{1}{\sqrt{2x-x^2}} = \frac{1}{\sqrt{1-(x-1)^2}}$$

$$\frac{x^2}{x^2+1} = 1 - \frac{1}{x^2+1}$$

$$\frac{2x}{x^2+2x+1} = \frac{2x+2-2}{x^2+2x+1}$$

$$= \frac{2x+2}{x^2+2x+1} - \frac{2}{(x+1)^2}$$

$$\cot^2 x = \csc^2 x - 1$$

$$\frac{1}{1+\sin x} = \left(\frac{1}{1+\sin x} \right) \left(\frac{1-\sin x}{1-\sin x} \right)$$

$$= \frac{1-\sin x}{1-\sin^2 x}$$

$$= \frac{1-\sin x}{\cos^2 x}$$

$$= \sec^2 x - \frac{\sin x}{\cos^2 x}$$