THEOREM 1.1   Some Basic Limits

Let b and c be real numbers, and let n be a positive integer.

1. $\lim\limits_{x \to c} b = b$ **2.** $\lim\limits_{x \to c} x = c$ **3.** $\lim\limits_{x \to c} x^n = c^n$

THEOREM 1.2   Properties of Limits

Let b and c be real numbers, let n be a positive integer, and let f and g be functions with the limits

$$\lim\limits_{x \to c} f(x) = L \quad \text{and} \quad \lim\limits_{x \to c} g(x) = K.$$

1. Scalar multiple: $\lim\limits_{x \to c} [b f(x)] = bL$

2. Sum or difference: $\lim\limits_{x \to c} [f(x) \pm g(x)] = L \pm K$

3. Product: $\lim\limits_{x \to c} [f(x)g(x)] = LK$

4. Quotient: $\lim\limits_{x \to c} \dfrac{f(x)}{g(x)} = \dfrac{L}{K}, \quad K \neq 0$

5. Power: $\lim\limits_{x \to c} [f(x)]^n = L^n$

A proof of this theorem is given in Appendix A.
See LarsonCalculus.com for Bruce Edwards's video of this proof.

EXAMPLE 2   The Limit of a Polynomial

Find the limit: $\lim\limits_{x \to 2} (4x^2 + 3)$.

THEOREM 1.3 **Limits of Polynomial and Rational Functions**

If p is a polynomial function and c is a real number, then

$$\lim_{x \to c} p(x) = p(c).$$

If r is a rational function given by $r(x) = p(x)/q(x)$ and c is a real number such that $q(c) \neq 0$, then

$$\lim_{x \to c} r(x) = r(c) = \frac{p(c)}{q(c)}.$$

EXAMPLE 3 **The Limit of a Rational Function**

Find the limit: $\displaystyle\lim_{x \to 1} \frac{x^2 + x + 2}{x + 1}$.

THEOREM 1.4   The Limit of a Function Involving a Radical

Let n be a positive integer. The limit below is valid for all c when n is odd, and is valid for $c > 0$ when n is even.

$$\lim_{x \to c} \sqrt[n]{x} = \sqrt[n]{c}$$

A proof of this theorem is given in Appendix A.
See LarsonCalculus.com for Bruce Edwards's video of this proof.

THEOREM 1.5   The Limit of a Composite Function

If f and g are functions such that $\lim_{x \to c} g(x) = L$ and $\lim_{x \to L} f(x) = f(L)$, then

$$\lim_{x \to c} f(g(x)) = f\left(\lim_{x \to c} g(x)\right) = f(L).$$

A proof of this theorem is given in Appendix A.
See LarsonCalculus.com for Bruce Edwards's video of this proof.

EXAMPLE 4   The Limit of a Composite Function

•••••▷ *See LarsonCalculus.com for an interactive version of this type of exam*

Find the limit.

a. $\lim_{x \to 0} \sqrt{x^2 + 4}$ **b.** $\lim_{x \to 3} \sqrt[3]{2x^2 - 10}$

THEOREM 1.6 Limits of Trigonometric Functions

Let c be a real number in the domain of the given trigonometric function.

1. $\lim\limits_{x \to c} \sin x = \sin c$ 2. $\lim\limits_{x \to c} \cos x = \cos c$ 3. $\lim\limits_{x \to c} \tan x = \tan c$

4. $\lim\limits_{x \to c} \cot x = \cot c$ 5. $\lim\limits_{x \to c} \sec x = \sec c$ 6. $\lim\limits_{x \to c} \csc x = \csc c$

THEOREM 1.7 Functions That Agree at All but One Point

Let c be a real number, and let $f(x) = g(x)$ for all $x \ne c$ in an open interval containing c. If the limit of $g(x)$ as x approaches c exists, then the limit of $f(x)$ also exists and

$$\lim\limits_{x \to c} f(x) = \lim\limits_{x \to c} g(x).$$

A proof of this theorem is given in Appendix A.

See LarsonCalculus.com for Bruce Edwards's video of this proof.

EXAMPLE 6   Finding the Limit of a Function

Find the limit.

$$\lim\limits_{x \to 1} \frac{x^3 - 1}{x - 1}$$

> **A Strategy for Finding Limits**

1. Learn to recognize which limits can be evaluated by direct substitution. (These limits are listed in Theorems 1.1 through 1.6.)

2. When the limit of $f(x)$ as x approaches c *cannot* be evaluated by direct substitution, try to find a function g that agrees with f for all x other than $x = c$. [Choose g such that the limit of $g(x)$ *can* be evaluated by direct substitution.] Then apply Theorem 1.7 to conclude *analytically* that

$$\lim_{x \to c} f(x) = \lim_{x \to c} g(x) = g(c).$$

3. Use a *graph* or *table* to reinforce your conclusion.

EXAMPLE 7 **Dividing Out Technique**

⋯▷ *See LarsonCalculus.com for an interactive version of this typ*

Find the limit: $\displaystyle\lim_{x \to -3} \frac{x^2 + x - 6}{x + 3}$.

EXAMPLE 8 **Rationalizing Technique**

Find the limit: $\lim\limits_{x \to 0} \dfrac{\sqrt{x + 1} - 1}{x}$.

THEOREM 1.8   The Squeeze Theorem

If $h(x) \le f(x) \le g(x)$ for all x in an open interval containing c, except possibly at c itself, and if

$$\lim_{x \to c} h(x) = L = \lim_{x \to c} g(x)$$

then $\lim_{x \to c} f(x)$ exists and is equal to L.

A proof of this theorem is given in Appendix A.

See LarsonCalculus.com for Bruce Edwards's video of this proof.

You can see the usefulness of the Squeeze Theorem (also called the Sandwich Theorem or the Pinching Theorem) in the proof of Theorem 1.9.

THEOREM 1.9   Two Special Trigonometric Limits

1. $\lim_{x \to 0} \dfrac{\sin x}{x} = 1$ 2. $\lim_{x \to 0} \dfrac{1 - \cos x}{x} = 0$

EXAMPLE 9 A Limit Involving a Trigonometric Function

Find the limit: $\lim\limits_{x \to 0} \dfrac{\tan x}{x}$.

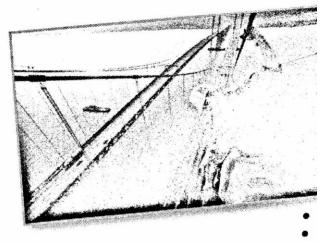**EXAMPLE 10** A Limit Involving a Trigonometric Function

Find the limit: $\lim\limits_{x \to 0} \dfrac{\sin 4x}{x}$.

• • Free-Falling Object • • • • • • • • • • • • • • • • •

In Exercises 101 and 102, use the position function $s(t) = -16t^2 + 500$, which gives the height (in feet) of an object that has fallen for t seconds from a height of 500 feet. The velocity at time $t = a$ seconds is given by

$$\lim_{t \to a} \frac{s(a) - s(t)}{a - t}.$$

101. A construction worker drops a full paint can from a height of 500 feet. How fast will the paint can be falling after 2 seconds?

102. A construction worker drops a full paint can from a height of 500 feet. When will the paint can hit the ground? At what velocity will the paint can impact the ground?