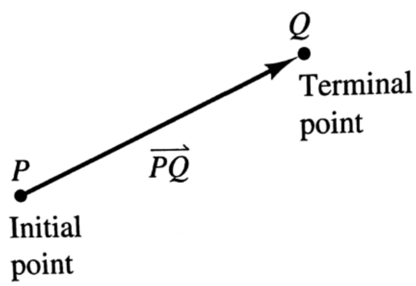
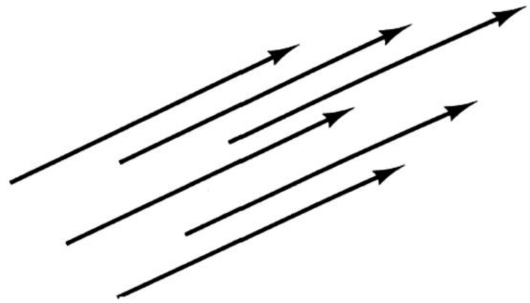


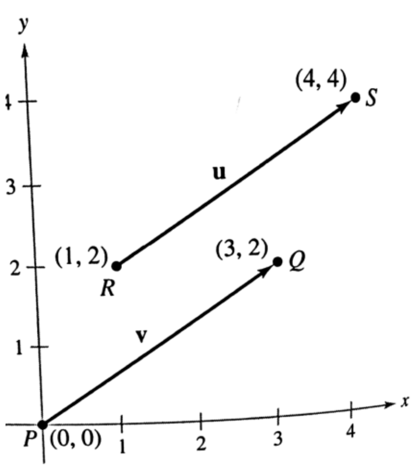
A directed line segment
Figure 11.1



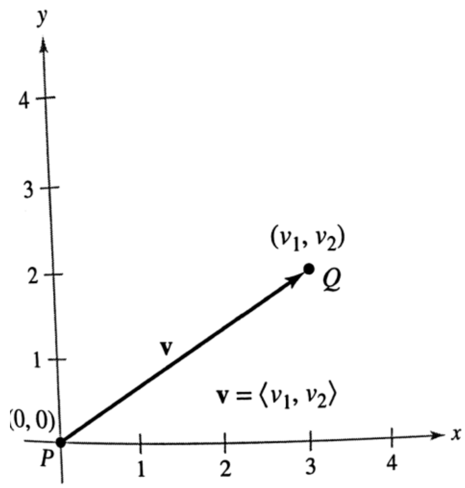
Equivalent directed line segments
Figure 11.2

EXAMPLE 1 **Vector Representation: Directed Line Segments**

Let **v** be represented by the directed line segment from $(0,0)$ to $(3,2)$, and let **u** be represented by the directed line segment from $(1,2)$ to $(4,4)$. Show that **v** and **u** are equivalent.



The vectors **u** and **v** are equivalent.
Figure 11.3



A vector in standard position
Figure 11.4

Definition of Component Form of a Vector in the Plane

If $\mathbf{v}$ is a vector in the plane whose initial point is the origin and whose terminal point is (v_1, v_2), then the **component form of v** is

$$\mathbf{v} = \langle v_1, v_2 \rangle.$$

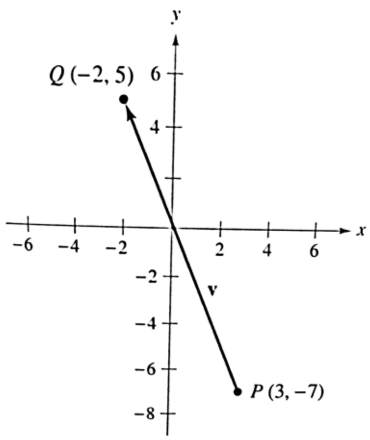
The coordinates v_1 and v_2 are called the **components of v.** If both the initial point and the terminal point lie at the origin, then $\mathbf{v}$ is called the **zero vector** and is denoted by $\mathbf{0} = \langle 0, 0 \rangle$.

$$\begin{aligned}\|\mathbf{v}\| &= \sqrt{(q_1 - p_1)^2 + (q_2 - p_2)^2} \\ &= \sqrt{v_1^2 + v_2^2}.\end{aligned}$$

Length of a vector

EXAMPLE 2 **Component Form and Length of a Vector**

Find the component form and length of the vector **v** that has initial point $(3, -7)$ and terminal point $(-2, 5)$.



Component form of **v**: $\mathbf{v} = \langle -5, 12 \rangle$
Figure 11.5

Vector Operations

Definitions of Vector Addition and Scalar Multiplication

Let $\mathbf{u} = \langle u_1, u_2 \rangle$ and $\mathbf{v} = \langle v_1, v_2 \rangle$ be vectors and let c be a scalar.

1. The **vector sum** of **u** and **v** is the vector $\mathbf{u} + \mathbf{v} = \langle u_1 + v_1, u_2 + v_2 \rangle$.
2. The **scalar multiple** of c and **u** is the vector

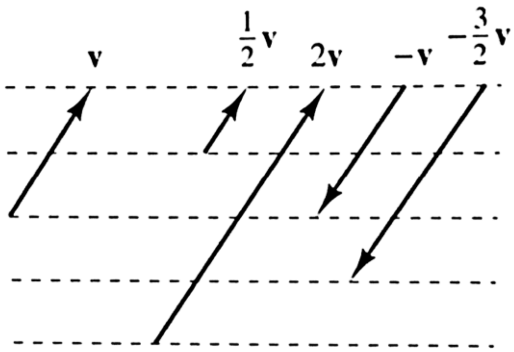
   $$c\mathbf{u} = \langle cu_1, cu_2 \rangle.$$

3. The **negative** of **v** is the vector

   $$-\mathbf{v} = (-1)\mathbf{v} = \langle -v_1, -v_2 \rangle.$$

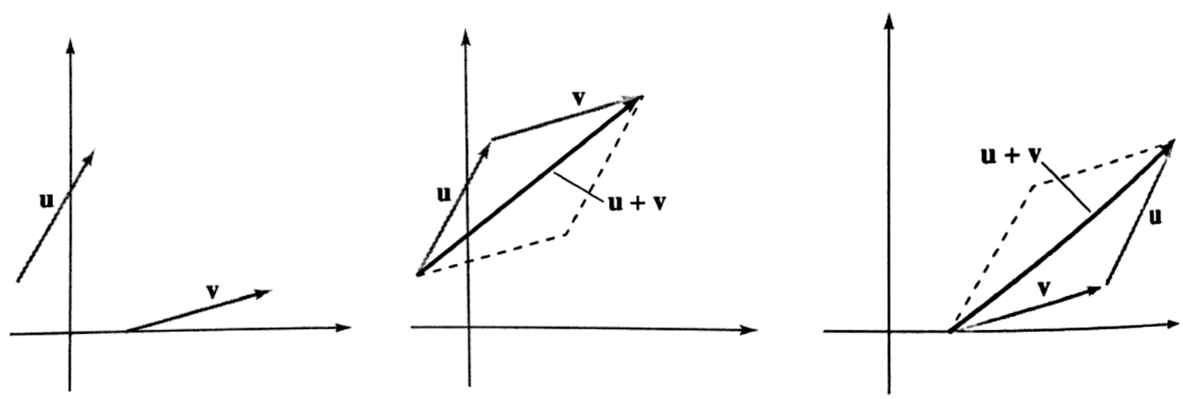
4. The **difference** of **u** and **v** is

   $$\mathbf{u} - \mathbf{v} = \mathbf{u} + (-\mathbf{v}) = \langle u_1 - v_1, u_2 - v_2 \rangle.$$



The scalar multiplication of **v**

Figure 11.6

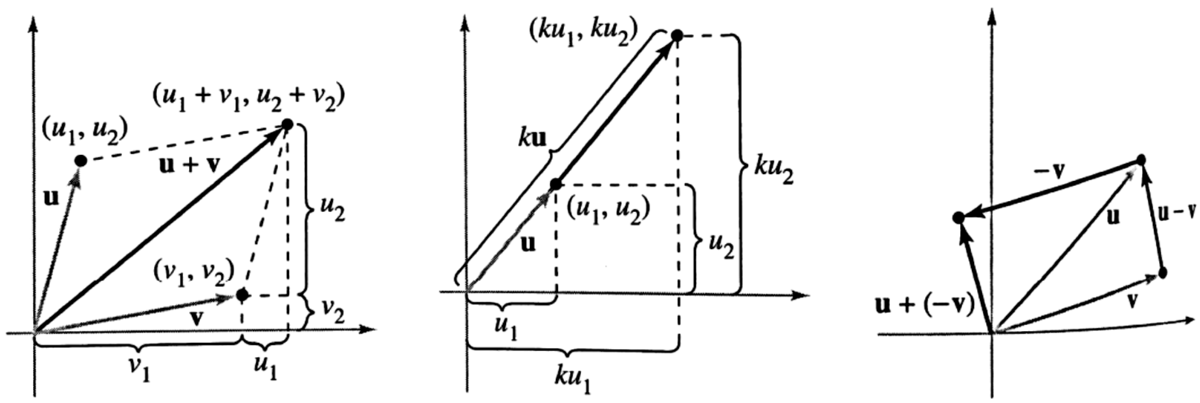


To find **u** + **v**,

(1) move the initial point of **v** to the terminal point of **u**, or

(2) move the initial point of **u** to the terminal point of **v**.

Figure 11.7



Vector addition   Scalar multiplication   Vector subtraction

Figure 11.8

The Granger Collection, New York

EXAMPLE 3 Vector Operations

For $\mathbf{v} = \langle -2, 5 \rangle$ and $\mathbf{w} = \langle 3, 4 \rangle$, find each of the vectors.

a. $\frac{1}{2}\mathbf{v}$ **b.** $\mathbf{w} - \mathbf{v}$ **c.** $\mathbf{v} + 2\mathbf{w}$

THEOREM 11.1 Properties of Vector Operations

Let $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ be vectors in the plane, and let c and d be scalars.

1. $\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}$ — Commutative Property
2. $(\mathbf{u} + \mathbf{v}) + \mathbf{w} = \mathbf{u} + (\mathbf{v} + \mathbf{w})$ — Associative Property
3. $\mathbf{u} + \mathbf{0} = \mathbf{u}$ — Additive Identity Property
4. $\mathbf{u} + (-\mathbf{u}) = \mathbf{0}$ — Additive Inverse Property
5. $c(d\mathbf{u}) = (cd)\mathbf{u}$
6. $(c + d)\mathbf{u} = c\mathbf{u} + d\mathbf{u}$ — Distributive Property
7. $c(\mathbf{u} + \mathbf{v}) = c\mathbf{u} + c\mathbf{v}$ — Distributive Property
8. $1(\mathbf{u}) = \mathbf{u}, 0(\mathbf{u}) = \mathbf{0}$

THEOREM 11.2 Length of a Scalar Multiple

Let $\mathbf{v}$ be a vector and let c be a scalar. Then

$$\|c\mathbf{v}\| = |c| \, \|\mathbf{v}\|.$$

$|c|$ is the absolute value of c.

THEOREM 11.3 Unit Vector in the Direction of v

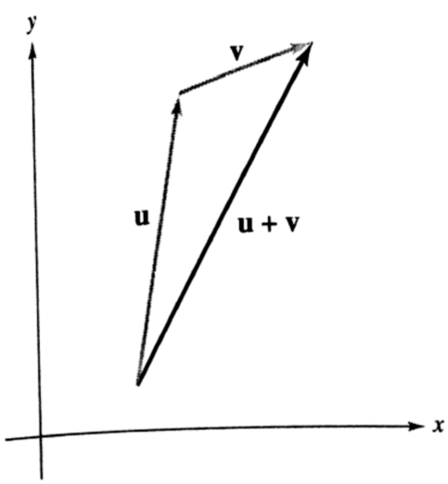
If $\mathbf{v}$ is a nonzero vector in the plane, then the vector

$$\mathbf{u} = \frac{\mathbf{v}}{\|\mathbf{v}\|} = \frac{1}{\|\mathbf{v}\|}\mathbf{v}$$

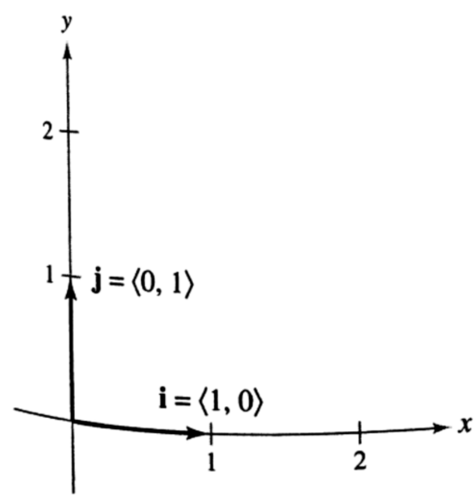
has length 1 and the same direction as $\mathbf{v}$.

EXAMPLE 4 Finding a Unit Vector

Find a unit vector in the direction of $\mathbf{v} = \langle -2, 5 \rangle$ and verify that it has length 1.



Triangle inequality
Figure 11.9



Standard unit vectors **i** and **j**
Figure 11.10

EXAMPLE 5 Writing a Linear Combination of Unit Vectors

Let **u** be the vector with initial point $(2, -5)$ and terminal point $(-1, 3)$, and let $\mathbf{v} = 2\mathbf{i} - \mathbf{j}$. Write each vector as a linear combination of **i** and **j**.

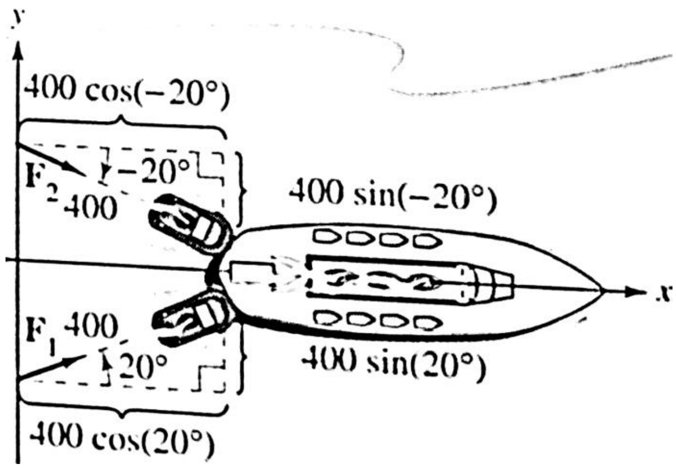
a. **u** **b.** $\mathbf{w} = 2\mathbf{u} - 3\mathbf{v}$

EXAMPLE 6 Writing a Vector of Given Magnitude and Direction

The vector **v** has a magnitude of 3 and makes an angle of $30° = \pi/6$ with the positive x-axis. Write **v** as a linear combination of the unit vectors **i** and **j**.

EXAMPLE 7 Finding the Resultant Force

Two tugboats are pushing an ocean liner, as shown in Figure 11.12. Each boat is exerting a force of 400 pounds. What is the resultant force on the ocean liner?

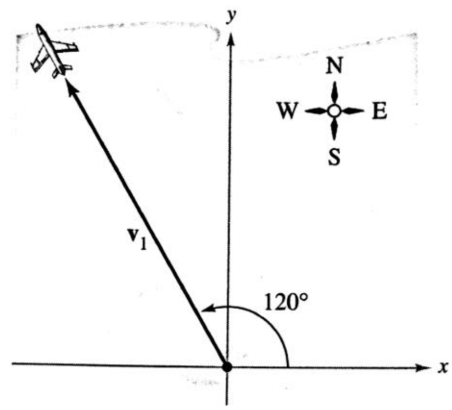


The resultant force on the ocean liner that is exerted by the two tugboats
Figure 11.12

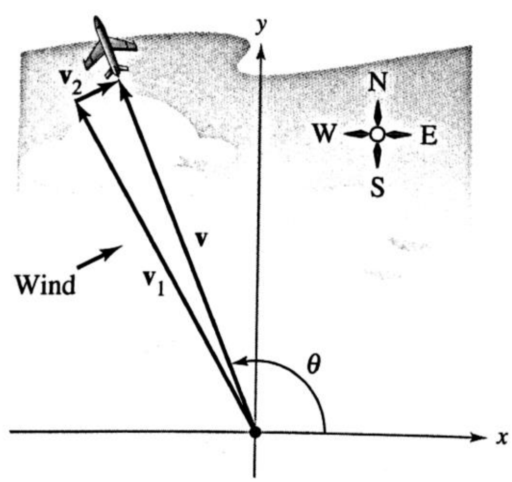
EXAMPLE 8 Finding a Velocity

See LarsonCalculus.com for an interactive version of this type of example.

An airplane is traveling at a fixed altitude with a negligible wind factor. The airplane is traveling at a speed of 500 miles per hour with a bearing of 330°, as shown in Figure 11.13(a). As the airplane reaches a certain point, it encounters wind with a velocity of 70 miles per hour in the direction N 45° E (45° east of north), as shown in Figure 11.13(b). What are the resultant speed and direction of the airplane?



(a) Direction without wind



(b) Direction with wind

Figure 11.13

75. Resultant Force Forces with magnitudes of 500 pounds and 200 pounds act on a machine part at angles of 30° and −45°, respectively, with the *x*-axis (see figure). Find the direction and magnitude of the resultant force.

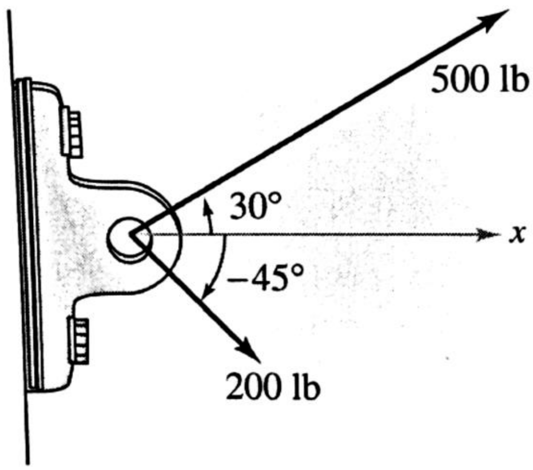
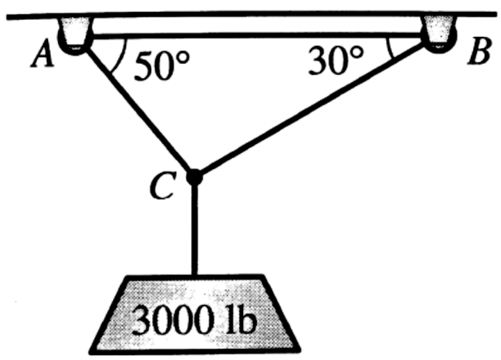


Figure for 75

80. Cable Tension Determine the tension in each cable supporting the given load for each figure.

(a)



(b)