Definition of Extrema

Let f be defined on an interval I containing c.

1. $f(c)$ is the **minimum of f on I** when $f(c) \leq f(x)$ for all x in I.
2. $f(c)$ is the **maximum of f on I** when $f(c) \geq f(x)$ for all x in I.

The minimum and maximum of a function on an interval are the **extreme values,** or **extrema** (the singular form of extrema is extremum), of the function on the interval. The minimum and maximum of a function on an interval are also called the **absolute minimum** and **absolute maximum,** or the **global minimum** and **global maximum,** on the interval. Extrema can occur at interior points or endpoints of an interval (see Figure 3.1). Extrema that occur at the endpoints are called **endpoint extrema.**

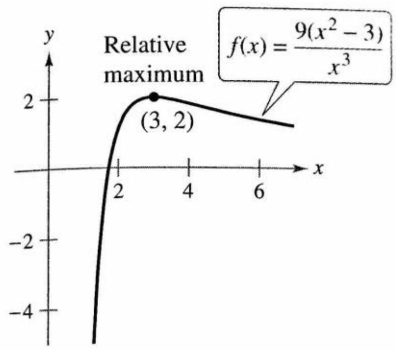
Definition of Relative Extrema

1. If there is an open interval containing c on which $f(c)$ is a maximum, then $f(c)$ is called a **relative maximum** of f, or you can say that f has a **relative maximum at $(c, f(c))$.**
2. If there is an open interval containing c on which $f(c)$ is a minimum, then $f(c)$ is called a **relative minimum** of f, or you can say that f has a **relative minimum at $(c, f(c))$.**

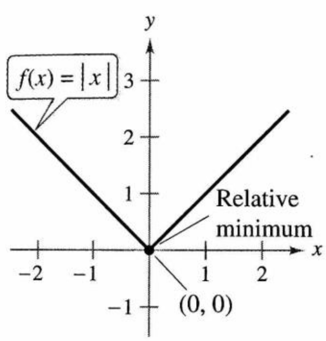
The plural of relative maximum is relative maxima, and the plural of relative minimum is relative minima. Relative maximum and relative minimum are sometimes called **local maximum** and **local minimum,** respectively.

EXAMPLE 1 The Value of the Derivative at Relative Extrema

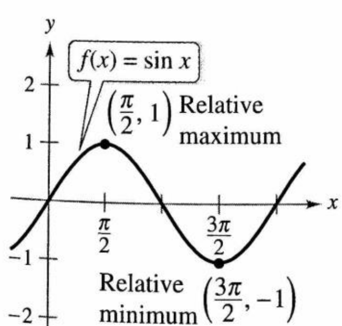
Find the value of the derivative at each relative extremum shown in Figure 3.3.



(a) $f'(3) = 0$



(b) $f'(0)$ does not exist.



(c) $f'\left(\frac{\pi}{2}\right) = 0; f'\left(\frac{3\pi}{2}\right) = 0$

Figure 3.3

Definition of a Critical Number

Let f be defined at c. If $f'(c) = 0$ or if f is not differentiable at c, then c is a **critical number** of f.

THEOREM 3.2 Relative Extrema Occur Only at Critical Numbers

If f has a relative minimum or relative maximum at $x = c$, then c is a critical number of f.

GUIDELINES FOR FINDING EXTREMA ON A CLOSED INTERVAL

To find the extrema of a continuous function f on a closed interval $[a, b]$, use these steps.

1. Find the critical numbers of f in (a, b).
2. Evaluate f at each critical number in (a, b).
3. Evaluate f at each endpoint of $[a, b]$.
4. The least of these values is the minimum. The greatest is the maximum.

EXAMPLE 2 **Finding Extrema on a Closed Interval**

Find the extrema of

$$f(x) = 3x^4 - 4x^3$$

on the interval $[-1, 2]$.

EXAMPLE 3 Finding Extrema on a Closed Interval

Find the extrema of $f(x) = 2x - 3x^{2/3}$ on the interval $[-1, 3]$.

EXAMPLE 4 Finding Extrema on a Closed Interval

See LarsonCalculus.com for an interactive version of this type of example.

Find the extrema of

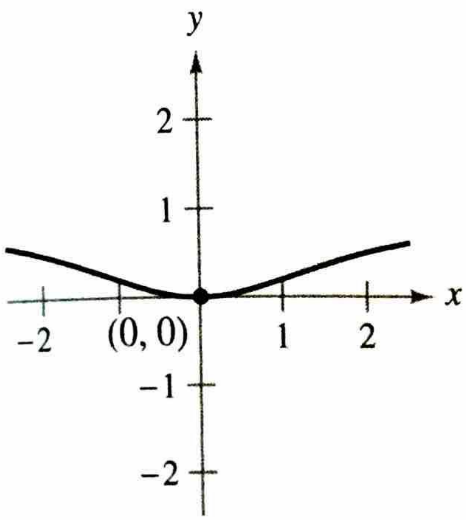
$$f(x) = 2 \sin x - \cos 2x$$

on the interval $[0, 2\pi]$.

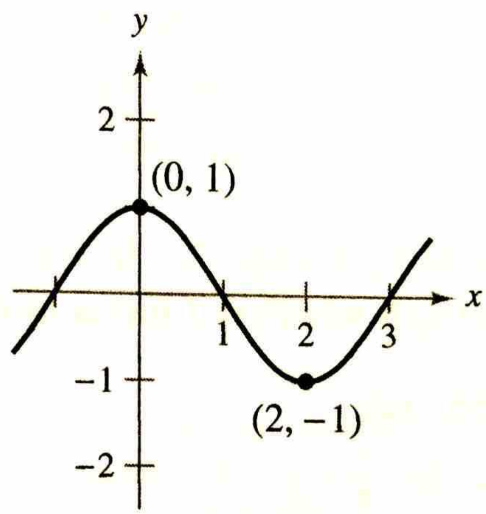
Finding the Value of the Derivative at Relative Extrema

In Exercises 1–6, find the value of the derivative (if it exists) at each indicated extremum.

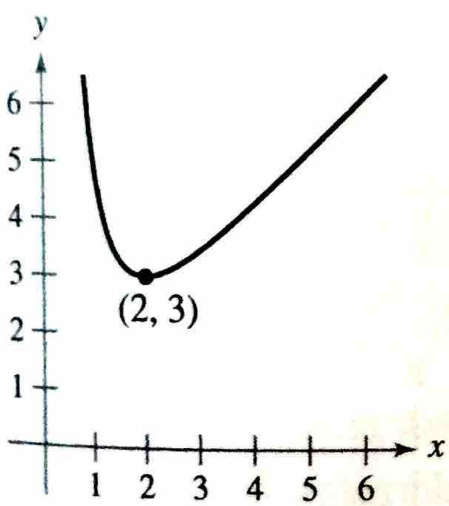
1. $f(x) = \dfrac{x^2}{x^2 + 4}$



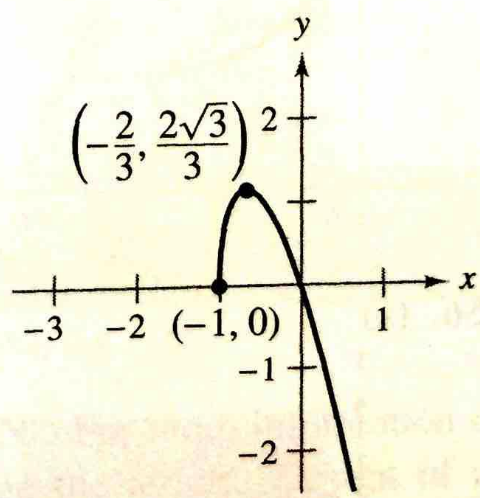
2. $f(x) = \cos \dfrac{\pi x}{2}$



3. $g(x) = x + \dfrac{4}{x^2}$



4. $f(x) = -3x\sqrt{x + 1}$



Solutions 1-4

Section 3.1 Extrema on an Interval

1. $f(x) = \dfrac{x^2}{x^2 + 4}$

$$f'(x) = \frac{(x^2 + 4)(2x) - (x^2)(2x)}{(x^2 + 4)^2} = \frac{8x}{(x^2 + 4)^2}$$

$f'(0) = 0$

2. $f(x) = \cos \dfrac{\pi x}{2}$

$$f'(x) = -\frac{\pi}{2} \sin \frac{\pi x}{2}$$

$f'(0) = 0$

$f'(2) = 0$

3. $f(x) = x + \dfrac{4}{x^2} = x + 4x^{-2}$

$$f'(x) = 1 - 8x^{-3} = 1 - \frac{8}{x^3}$$

$f'(2) = 0$

4. $f(x) = -3x\sqrt{x + 1}$

$$\begin{aligned} f'(x) &= -3x\left[\tfrac{1}{2}(x + 1)^{-1/2}\right] + \sqrt{x + 1}(-3) \\ &= -\tfrac{3}{2}(x + 1)^{-1/2}\left[x + 2(x + 1)\right] \\ &= -\tfrac{3}{2}(x + 1)^{-1/2}(3x + 2) \end{aligned}$$

$f'\left(-\tfrac{2}{3}\right) = 0$