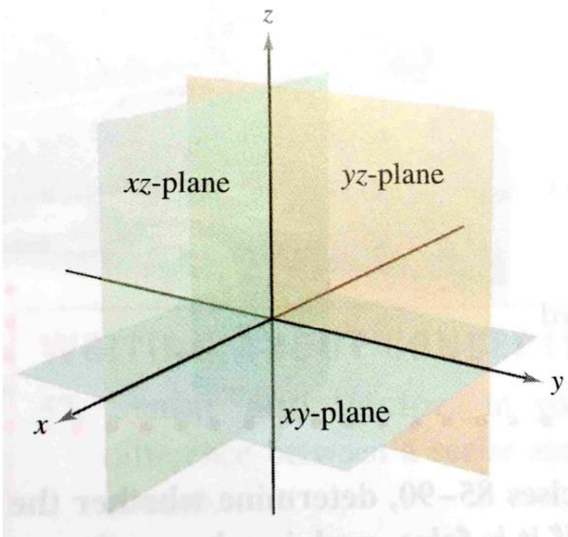
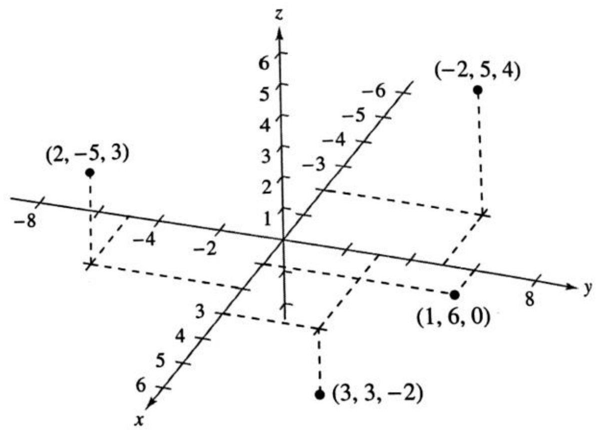
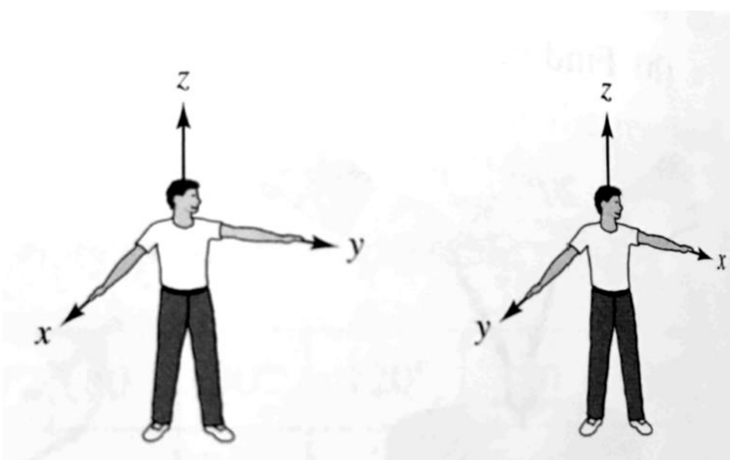


The three-dimensional coordinate system
Figure 11.14



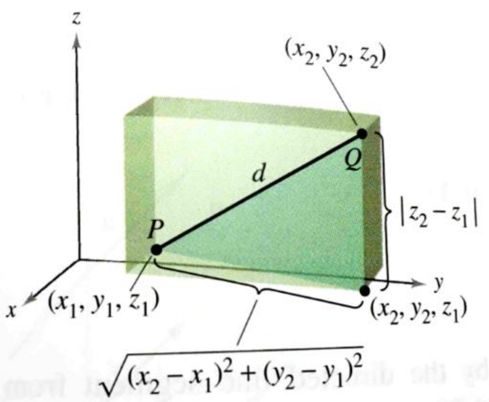
Points in the three-dimensional coordinate system are represented by ordered triples.
Figure 11.15



Right-handed system

Left-handed system

Figure 11.16



The distance between two points in space

Figure 11.17

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$ Distance Formula

EXAMPLE 1 **Finding the Distance Between Two Points in Space**

Find the distance between the points $(2, -1, 3)$ and $(1, 0, -2)$.

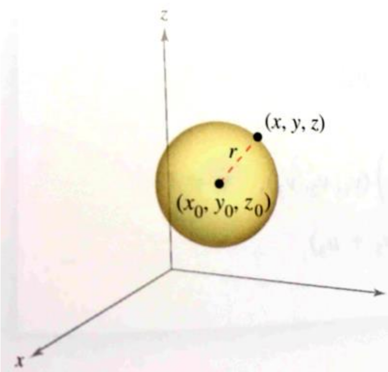


Figure 11.18

EXAMPLE 2 Finding the Equation of a Sphere

Find the standard equation of the sphere that has the points

$$(5, -2, 3) \quad \text{and} \quad (0, 4, -3)$$

as endpoints of a diameter.

Vectors in Space

Let $\mathbf{u} = \langle u_1, u_2, u_3 \rangle$ and $\mathbf{v} = \langle v_1, v_2, v_3 \rangle$ be vectors in space and let c be a scalar.

1. *Equality of Vectors:* $\mathbf{u} = \mathbf{v}$ if and only if $u_1 = v_1$, $u_2 = v_2$, and $u_3 = v_3$.
2. *Component Form:* If $\mathbf{v}$ is represented by the directed line segment from $P(p_1, p_2, p_3)$ to $Q(q_1, q_2, q_3)$, then
$$\mathbf{v} = \langle v_1, v_2, v_3 \rangle = \langle q_1 - p_1, q_2 - p_2, q_3 - p_3 \rangle.$$
3. *Length:* $\|\mathbf{v}\| = \sqrt{v_1^2 + v_2^2 + v_3^2}$
4. *Unit Vector in the Direction of* $\mathbf{v}$: $\dfrac{\mathbf{v}}{\|\mathbf{v}\|} = \left(\dfrac{1}{\|\mathbf{v}\|}\right)\langle v_1, v_2, v_3 \rangle, \quad \mathbf{v} \neq \mathbf{0}$
5. *Vector Addition:* $\mathbf{v} + \mathbf{u} = \langle v_1 + u_1, v_2 + u_2, v_3 + u_3 \rangle$
6. *Scalar Multiplication:* $c\mathbf{v} = \langle cv_1, cv_2, cv_3 \rangle$

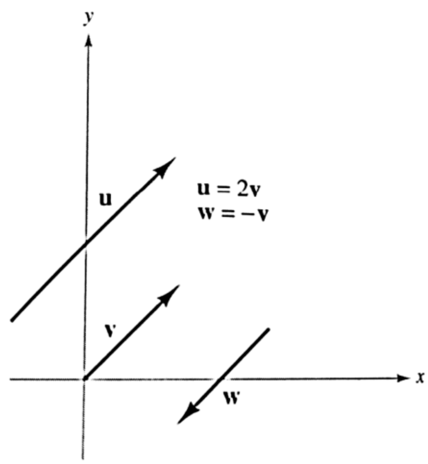
EXAMPLE 3 **Finding the Component Form of a Vector in Space**

See LarsonCalculus.com for an interactive version of this type of example.

Find the component form and magnitude of the vector $\mathbf{v}$ having initial point $(-2, 3, 1)$ and terminal point $(0, -4, 4)$. Then find a unit vector in the direction of $\mathbf{v}$.

Definition of Parallel Vectors

Two nonzero vectors $\mathbf{u}$ and $\mathbf{v}$ are **parallel** when there is some scalar c such that $\mathbf{u} = c\mathbf{v}$.



Parallel vectors
Figure 11.21

EXAMPLE 4 Parallel Vectors

Vector **w** has initial point $(2, -1, 3)$ and terminal point $(-4, 7, 5)$. Which of the following vectors is parallel to **w**?

a. $\mathbf{u} = \langle 3, -4, -1 \rangle$

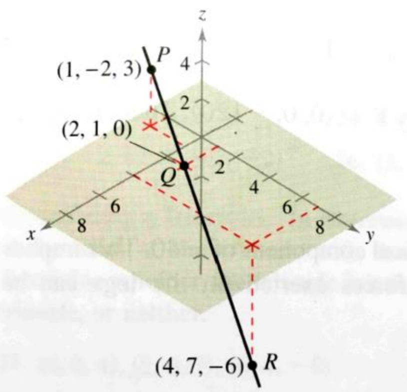
b. $\mathbf{v} = \langle 12, -16, 4 \rangle$

EXAMPLE 5 Using Vectors to Determine Collinear Points

Determine whether the points

$$P(1, -2, 3), \quad Q(2, 1, 0), \quad \text{and} \quad R(4, 7, -6)$$

are collinear.



The points P, Q, and R lie on the same line.

Figure 11.22

EXAMPLE 6 **Standard Unit Vector Notation**

a. Write the vector $\mathbf{v} = 4\mathbf{i} - 5\mathbf{k}$ in component form.

b. Find the terminal point of the vector $\mathbf{v} = 7\mathbf{i} - \mathbf{j} + 3\mathbf{k}$, given that the initial point is $P(-2, 3, 5)$.

c. Find the magnitude of the vector $\mathbf{v} = -6\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$. Then find a unit vector in the direction of $\mathbf{v}$.

EXAMPLE 7 **Measuring Force**

A television camera weighing 120 pounds is supported by a tripod, as shown in Figure 11.23. Represent the force exerted on each leg of the tripod as a vector.

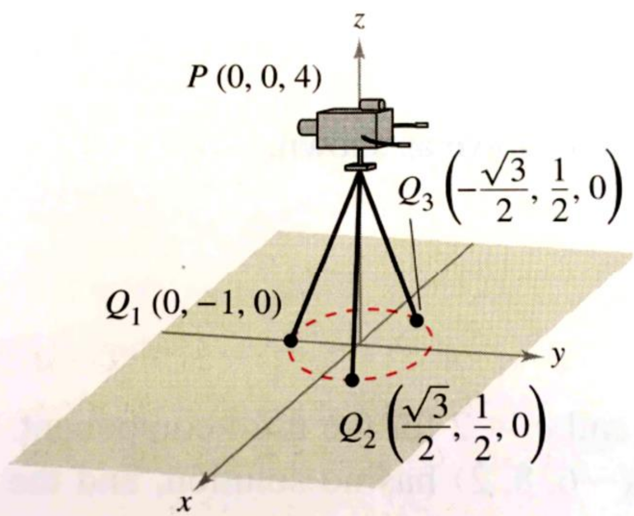


Figure 11.23