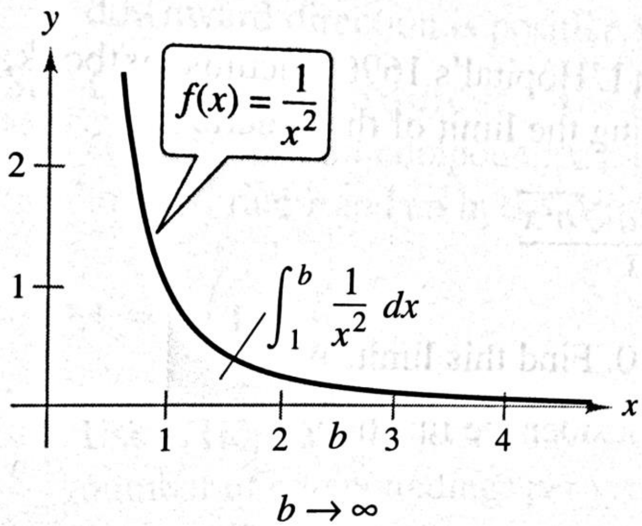


The unbounded region has an area of

Figure 8.17

Definition of Improper Integrals with Infinite Integration Limits

1. If f is continuous on the interval $[a, \infty)$, then

$$\int_a^{\infty} f(x)\,dx = \lim_{b\to\infty} \int_a^b f(x)\,dx.$$

2. If f is continuous on the interval $(-\infty, b]$, then

$$\int_{-\infty}^{b} f(x)\,dx = \lim_{a\to-\infty} \int_a^b f(x)\,dx.$$

3. If f is continuous on the interval $(-\infty, \infty)$, then

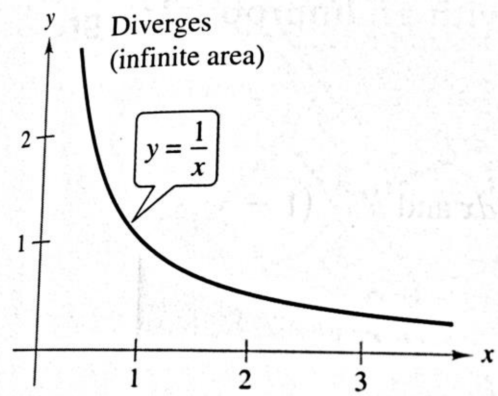
$$\int_{-\infty}^{\infty} f(x)\,dx = \int_{-\infty}^{c} f(x)\,dx + \int_{c}^{\infty} f(x)\,dx$$

where c is any real number (see Exercise 111).

In the first two cases, the improper integral **converges** when the limit exists—otherwise, the improper integral **diverges**. In the third case, the improper integral on the left diverges when either of the improper integrals on the right diverges.

EXAMPLE 1 An Improper Integral That Diverges

Evaluate $\int_1^\infty \frac{dx}{x}$.



This unbounded region has an infinite area.

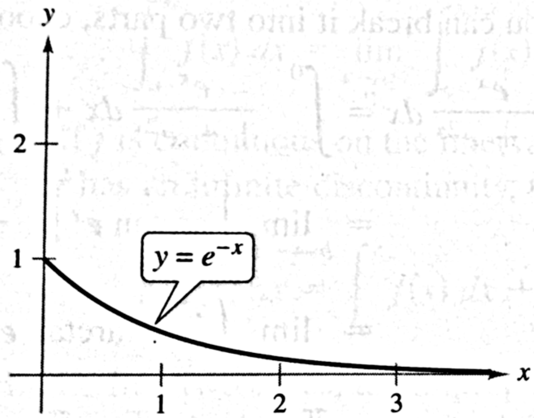
Figure 8.18

EXAMPLE 2 **Improper Integrals That Converge**

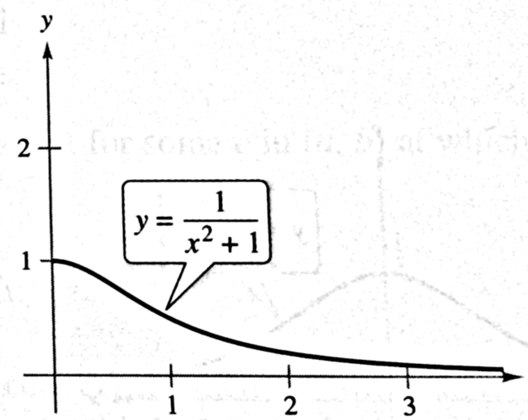
Evaluate each improper integral.

a. $\int_0^\infty e^{-x}\,dx$

b. $\int_0^\infty \frac{1}{x^2+1}\,dx$



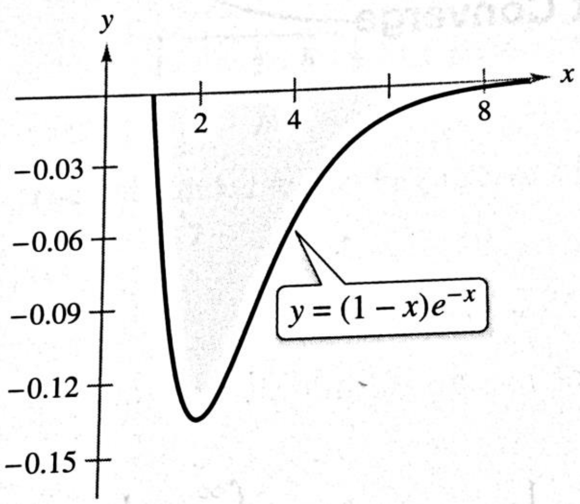
The area of the unbounded region is 1.
Figure 8.19



The area of the unbounded region is $\pi/2$.
Figure 8.20

EXAMPLE 3 **Using L'Hôpital's Rule with an Improper Integral**

Evaluate $\int_1^{\infty} (1 - x)e^{-x}\, dx$.

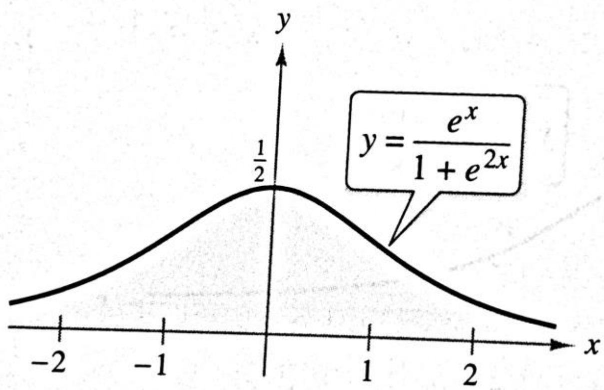


The area of the unbounded region is $|-1/e|$.

Figure 8.21

EXAMPLE 4 **Infinite Upper and Lower Limits of Integration**

Evaluate $\displaystyle\int_{-\infty}^{\infty} \frac{e^x}{1 + e^{2x}}\,dx.$



The area of the unbounded region is $\pi/2$.

Figure 8.22

EXAMPLE 5 **Sending a Space Module into Orbit**

In Example 3 in Section 7.5, you found that it would require 10,000 mile-tons of work to propel a 15-metric-ton space module to a height of 800 miles above Earth. How much work is required to propel the module an unlimited distance away from Earth's surface?

The work required to move a 15-metric-ton space module an unlimited distance away from Earth is about 6.984×10^{11} foot-pounds.

Definition of Improper Integrals with Infinite Discontinuities

1. If f is continuous on the interval $[a, b)$ and has an infinite discontinuity at b, then

$$\int_a^b f(x)\,dx = \lim_{c \to b^-} \int_a^c f(x)\,dx.$$

2. If f is continuous on the interval $(a, b]$ and has an infinite discontinuity at a, then

$$\int_a^b f(x)\,dx = \lim_{c \to a^+} \int_c^b f(x)\,dx.$$

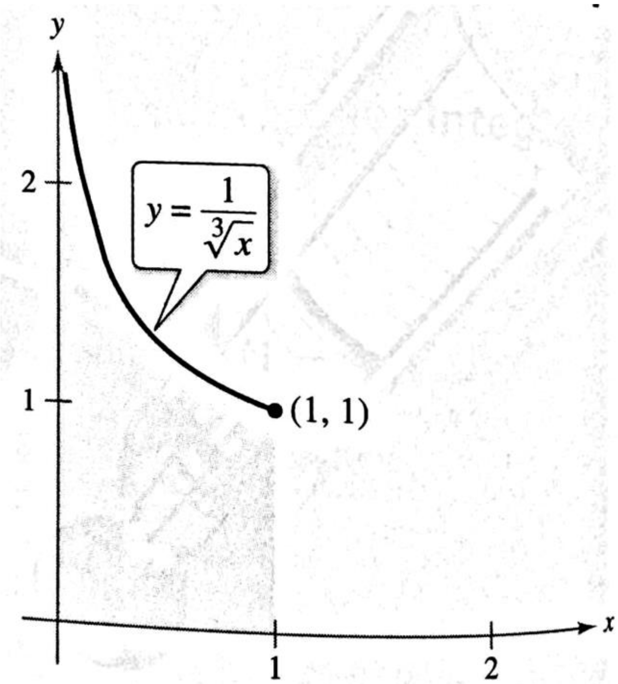
3. If f is continuous on the interval $[a, b]$, except for some c in (a, b) at which f has an infinite discontinuity, then

$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx.$$

In the first two cases, the improper integral **converges** when the limit exists—otherwise, the improper integral **diverges**. In the third case, the improper integral on the left diverges when either of the improper integrals on the right diverges.

EXAMPLE 6 **An Improper Integral with**

Evaluate $\int_0^1 \frac{dx}{\sqrt[3]{x}}$.



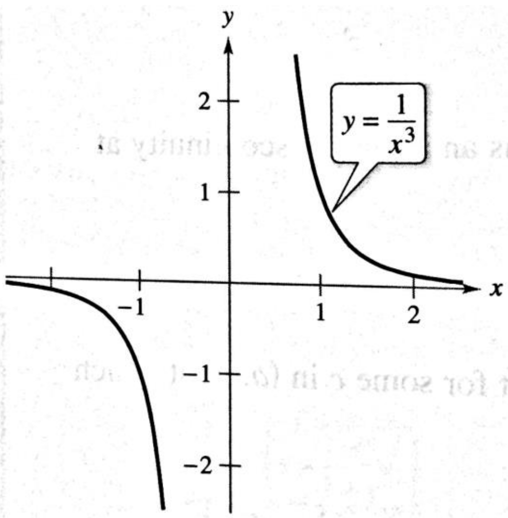
Infinite discontinuity at $x = 0$
Figure 8.23

EXAMPLE 7 **An Improper Integral T**

Evaluate $\int_0^2 \frac{dx}{x^3}$.

EXAMPLE 8 **An Improper Integral with an**

Evaluate $\int_{-1}^{2} \frac{dx}{x^3}$.



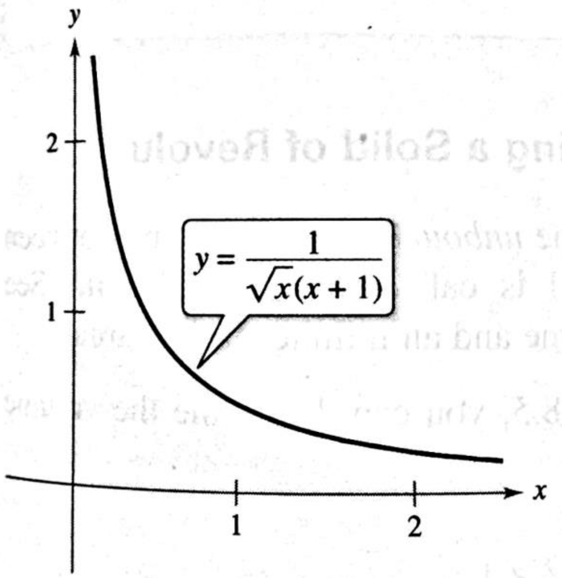
The improper integral $\int_{-1}^{2} \frac{dx}{x^3}$ diverges.

Figure 8.24

EXAMPLE 9 **A Doubly Improper Integral**

See LarsonCalculus.com for an interactive version of this type of examp

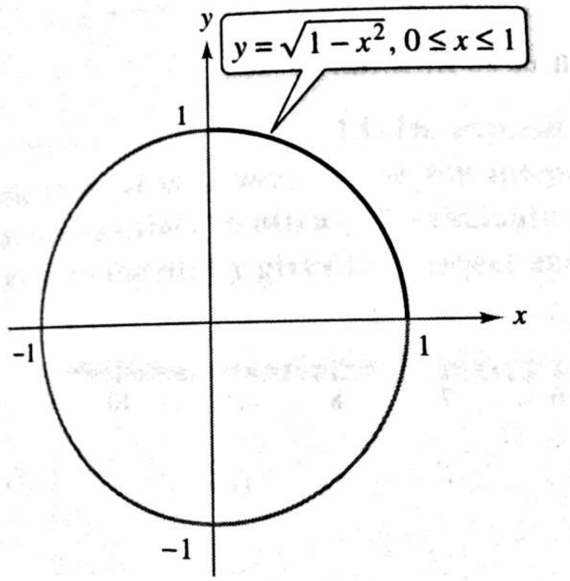
Evaluate $\displaystyle\int_0^{\infty} \frac{dx}{\sqrt{x}(x+1)}$.



The area of the unbounded region is π.
Figure 8.25

EXAMPLE 10 **An Application Involving Arc Length**

Use the formula for arc length to show that the circumference of the circle $x^2 + y^2 = 1$ is 2π.



The circumference of the circle is 2π.
Figure 8.26