EXAMPLE 1 Using the Product Rule

Find the derivative of $h(x) = (3x - 2x^2)(5 + 4x)$.

EXAMPLE 2 Using the Product Rule

Find the derivative of $y = 3x^2 \sin x$.

EXAMPLE 3 Using the Product Rule

Find the derivative of $y = 2x\cos x - 2\sin x$.

Using the Product Rule **In Exercises 1–6, use the Product Rule to find the derivative of the function.**

1. $g(x) = (x^2 + 3)(x^2 - 4x)$
2. $y = (3x - 4)(x^3 + 5)$
3. $h(t) = \sqrt{t}(1 - t^2)$
4. $g(s) = \sqrt{s}(s^2 + 8)$
5. $f(x) = x^3 \cos x$
6. $g(x) = \sqrt{x}\sin x$

EXAMPLE 4 Using the Quotient Rule

Find the derivative of $y = \dfrac{5x - 2}{x^2 + 1}$.

EXAMPLE 5 Rewriting Before Differentiating

Find an equation of the tangent line to the graph of $f(x) = \dfrac{3 - (1/x)}{x + 5}$ at $(-1, 1)$.

Using the Quotient Rule **In Exercises 7–12, use the Quotient Rule to find the derivative of the function.**

7. $f(x) = \dfrac{x}{x^2 + 1}$

8. $g(t) = \dfrac{3t^2 - 1}{2t + 5}$

9. $h(x) = \dfrac{\sqrt{x}}{x^3 + 1}$

10. $f(x) = \dfrac{x^2}{2\sqrt{x} + 1}$

11. $g(x) = \dfrac{\sin x}{x^2}$

12. $f(t) = \dfrac{\cos t}{t^3}$

EXAMPLE 6 **Using the Constant Multiple Rule**

Original Function	Rewrite	Differentiate	Simplify
a. $y = \dfrac{x^2 + 3x}{6}$	$y = \dfrac{1}{6}(x^2 + 3x)$	$y' = \dfrac{1}{6}(2x + 3)$	$y' = \dfrac{2x + 3}{6}$
b. $y = \dfrac{5x^4}{8}$	$y = \dfrac{5}{8}x^4$	$y' = \dfrac{5}{8}(4x^3)$	$y' = \dfrac{5}{2}x^3$
c. $y = \dfrac{-3(3x - 2x^2)}{7x}$	$y = -\dfrac{3}{7}(3 - 2x)$	$y' = -\dfrac{3}{7}(-2)$	$y' = \dfrac{6}{7}$
d. $y = \dfrac{9}{5x^2}$	$y = \dfrac{9}{5}(x^{-2})$	$y' = \dfrac{9}{5}(-2x^{-3})$	$y' = -\dfrac{18}{5x^3}$

Derivatives of Trigonometric Functions

Knowing the derivatives of the sine and cosine functions, you can use the Quoti
to find the derivatives of the four remaining trigonometric functions.

THEOREM 2.9 Derivatives of Trigonometric Functions

$$\frac{d}{dx}[\tan x] = \sec^2 x \qquad \frac{d}{dx}[\cot x] = -\csc^2 x$$

$$\frac{d}{dx}[\sec x] = \sec x \tan x \qquad \frac{d}{dx}[\csc x] = -\csc x \cot x$$

EXAMPLE 8

▷ *See LarsonCa*

Function

a. $y = x - \tan x$

b. $y = x \sec x$

• ▷ **EXAMPLE 9** **Different Forms of a Derivative**

Differentiate both forms of

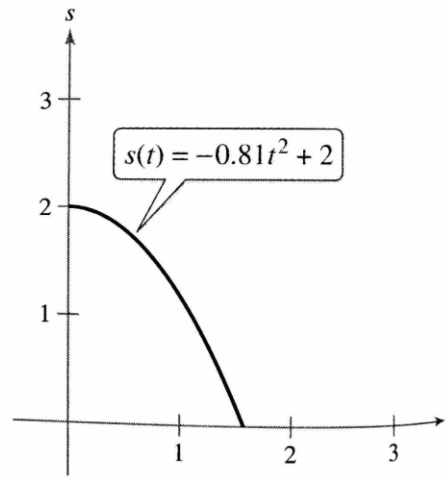
s.
$$y = \frac{1 - \cos x}{\sin x} = \csc x - \cot x.$$

EXAMPLE 10 **Finding the Acceleration Due to Gravity**

Because the moon has no atmosphere, a falling object on the moon encounters no air resistance. In 1971, astronaut David Scott demonstrated that a feather and a hammer fall at the same rate on the moon. The position function for each of these falling objects is

$$s(t) = -0.81t^2 + 2$$

where $s(t)$ is the height in meters and t is the time in seconds, as shown in the figure at the right. What is the ratio of Earth's gravitational force to the moon's?



Evaluating a Derivative In Exercises 59–62, evaluate the derivative of the function at the given point. Use a graphing utility to verify your result.

Function	Point
59. $y = \dfrac{1 + \csc x}{1 - \csc x}$	$\left(\dfrac{\pi}{6}, -3\right)$
60. $f(x) = \tan x \cot x$	$(1, 1)$

AP **Finding an Equation of a Tangent Line** In Exercises 63–68, (a) find an equation of the tangent line to the graph of f at the given point, (b) use a graphing utility to graph the function and its tangent line at the point, and (c) use the *derivative* feature of a graphing utility to confirm your results.

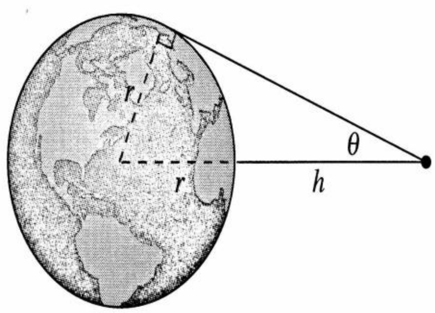
67. $f(x) = \tan x, \quad \left(\dfrac{\pi}{4}, 1\right)$

Horizontal Tangent Line **In Exercises 73–76, determine the point(s) at which the graph of the function has a horizontal tangent line.**

73. $f(x) = \dfrac{2x - 1}{x^2}$

74. $f(x) = \dfrac{x^2}{x^2 + 1}$

90. Satellites When satellites observe Earth, they can scan only part of Earth's surface. Some satellites have sensors that can measure the angle θ shown in the figure. Let h represent the satellite's distance from Earth's surface, and let r represent Earth's radius.



(a) Show that $h = r(\csc \theta - 1)$.

(b) Find the rate at which h is changing with respect to θ when $\theta = 30°$. (Assume $r = 3960$ miles.)

... Quotient Rules a

1. $g(x) = (x^2 + 3)(x^2 - 4x)$

$$g'(x) = (x^2 + 3)(2x - 4) + (x^2 - 4x)(2x)$$
$$= 2x^3 - 4x^2 + 6x - 12 + 2x^3 - 8x^2$$
$$= 4x^3 - 12x^2 + 6x - 12$$
$$= 2(2x^3 - 6x^2 + 3x - 6)$$

2. $y = (3x - 4)(x^3 + 5)$

$$y' = (3x - 4)(3x^2) + (x^3 + 5)(3)$$
$$= 9x^3 - 12x^2 + 3x^3 + 15$$
$$= 12x^3 - 12x^2 + 15$$

3. $h(t) = \sqrt{t}(1 - t^2) = t^{1/2}(1 - t^2)$

$$h'(t) = t^{1/2}(-2t) + (1 - t^2)\frac{1}{2}t^{-1/2}$$
$$= -2t^{3/2} + \frac{1}{2t^{1/2}} - \frac{1}{2}t^{3/2}$$
$$= -\frac{5}{2}t^{3/2} + \frac{1}{2t^{1/2}}$$
$$= \frac{1 - 5t^2}{2t^{1/2}} = \frac{1 - 5t^2}{2\sqrt{t}}$$

4. $g(s) = \sqrt{s}(s^2 + 8) = s^{1/2}(s^2 + 8)$

$$g'(s) = s^{1/2}(2s) + (s^2 + 8)\frac{1}{2}s^{-1/2}$$

$$= 2s^{3/2} + \frac{1}{2}s^{3/2} + 4s^{-1/2}$$

$$= \frac{5}{2}s^{3/2} + \frac{4}{s^{1/2}}$$

$$= \frac{5s^2 + 8}{2\sqrt{s}}$$

5. $f(x) = x^3 \cos x$

$$f'(x) = x^3(-\sin x) + \cos x(3x^2)$$

$$= 3x^2 \cos x - x^3 \sin x$$

$$= x^2(3\cos x - x\sin x)$$

6. $g(x) = \sqrt{x} \sin x$

$$g'(x) = \sqrt{x}\cos x + \sin x\left(\frac{1}{2\sqrt{x}}\right)$$

$$= \sqrt{x}\cos x + \frac{1}{2\sqrt{x}}\sin x$$

4. $g(s) = \sqrt{s}\left(s^2 + 8\right) = s^{1/2}\left(s^2 + 8\right)$

$$g'(s) = s^{1/2}(2s) + \left(s^2 + 8\right)\frac{1}{2}s^{-1/2}$$

$$= 2s^{3/2} + \frac{1}{2}s^{3/2} + 4s^{-1/2}$$

$$= \frac{5}{2}s^{3/2} + \frac{4}{s^{1/2}}$$

$$= \frac{5s^2 + 8}{2\sqrt{s}}$$

5. $f(x) = x^3 \cos x$

$$f'(x) = x^3(-\sin x) + \cos x\left(3x^2\right)$$

$$= 3x^2 \cos x - x^3 \sin x$$

$$= x^2(3 \cos x - x \sin x)$$

6. $g(x) = \sqrt{x} \sin x$

$$g'(x) = \sqrt{x} \cos x + \sin x\left(\frac{1}{2\sqrt{x}}\right)$$

$$= \sqrt{x} \cos x + \frac{1}{2\sqrt{x}} \sin x$$

10. $f(x) = \dfrac{x^2}{2\sqrt{x} + 1}$

$$f'(x) = \frac{(2\sqrt{x} + 1)(2x) - x^2(x^{-1/2})}{(2\sqrt{x} + 1)^2}$$

$$= \frac{4x^{3/2} + 2x - x^{3/2}}{(2\sqrt{x} + 1)^2}$$

$$= \frac{3x^{3/2} + 2x}{(2\sqrt{x} + 1)^2}$$

$$= \frac{x(3\sqrt{x} + 2)}{(2\sqrt{x} + 1)^2}$$

11. $g(x) = \dfrac{\sin x}{x^2}$

$$g'(x) = \frac{x^2(\cos x) - \sin x(2x)}{(x^2)^2} = \frac{x \cos x - 2 \sin x}{x^3}$$

12. $f(t) = \dfrac{\cos t}{t^3}$

$$f'(t) = \frac{t^3(-\sin t) - \cos t(3t^2)}{(t^3)^2} = -\frac{t \sin t + 3 \cos t}{t^4}$$