$$\text{Force} = (\text{mass})(\text{acceleration}).$$

: below lists some commonly used measures of mass and force, togeth
version factors.

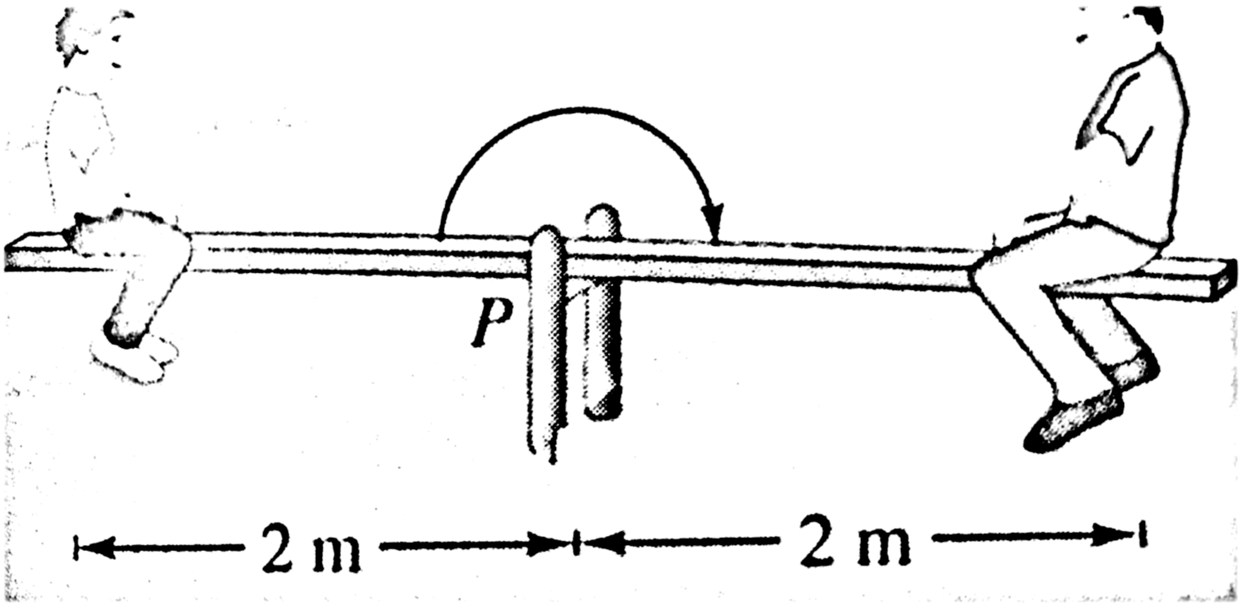
System of Measurement	Measure of Mass	Measure of Force
U.S.	Slug	Pound = (slug)(ft/sec^2)
International	Kilogram	Newton = (kilogram)(m/sec^2)
C-G-S	Gram	Dyne = (gram)(cm/sec^2)

Conversions:

1 pound = 4.448 newtons	1 slug = 14.59 kilograms
1 newton = 0.2248 pound	1 kilogram = 0.06852 slug
1 dyne = 0.000002248 pound	1 gram = 0.00006852 slug
1 dyne = 0.00001 newton	1 foot = 0.3048 meter

EXAMPLE 1 Mass on the Surface of Earth

Find the mass (in slugs) of an object whose weight at sea level is 1 pound.



Moments and Center of Mass: One-Dimensional System

Let the point masses $m_1, m_2, \ldots, m_n$ be located at $x_1, x_2, \ldots, x_n$.

1. The **moment about the origin** is

$$M_0 = m_1x_1 + m_2x_2 + \cdots + m_nx_n.$$

2. The **center of mass** is

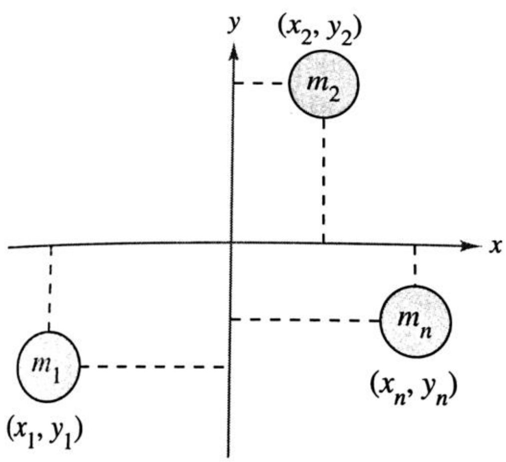
$$\bar{x} = \frac{M_0}{m}$$

where $m = m_1 + m_2 + \cdots + m_n$ is the **total mass** of the system.

EXAMPLE 2 **The Center of Mass of a Linear System**

Find the center of mass of the linear system shown in Figure 7.55.

Figure 7.55



In a two-dimensional system, there is a moment about the y-axis M_y and a moment about the x-axis M_x.

Figure 7.56

Moment and Center of Mass: Two-Dimensional System

Let the point masses $m_1, m_2, \ldots, m_n$ be located at $(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)$.

1. The **moment about the y-axis** is

$$M_y = m_1x_1 + m_2x_2 + \ldots m_nx_n.$$

2. The **moment about the x-axis** is

$$M_x = m_1y_1 + m_2y_2 + \ldots m_ny_n.$$

3. The **center of mass** $(\bar{x}, \bar{y})$ (or **center of gravity**) is

$$\bar{x} = \frac{M_y}{m} \quad \text{and} \quad \bar{y} = \frac{M_x}{m}$$

where

$$m = m_1 + m_2 + \ldots + m_n$$

is the **total mass** of the system.

EXAMPLE 3 The Center of Mass of a Two-Dimensional System

Find the center of mass of a system of point masses $m_1 = 6, m_2 = 3, m_3 = 2$, and $m_4 = 9$, located at

$$(3, -2), \quad (0, 0), \quad (-5, 3), \quad \text{and} \quad (4, 2)$$

as shown in Figure 7.57.

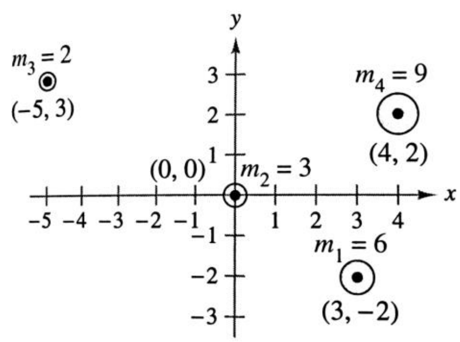
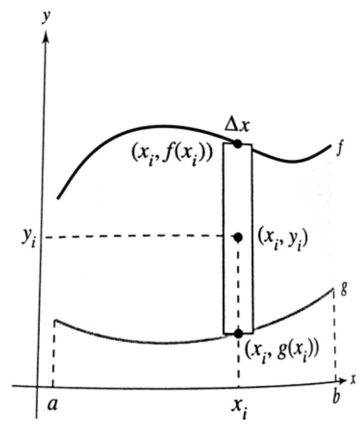


Figure 7.57



Planar lamina of uniform density ρ

Figure 7.59

Moments and Center of Mass of a Planar Lamina

Let f and g be continuous functions such that $f(x) \geq g(x)$ on $[a, b]$, and consider the planar lamina of uniform density ρ bounded by the graphs of $y = f(x)$, $y = g(x)$, and $a \leq x \leq b$.

1. The **moments about the x- and y-axes** are

$$M_x = \rho \int_a^b \left[\frac{f(x) + g(x)}{2}\right][f(x) - g(x)]\,dx$$

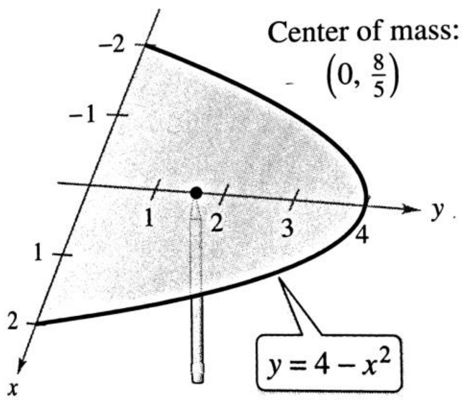
$$M_y = \rho \int_a^b x[f(x) - g(x)]\,dx.$$

2. The **center of mass** $(\bar{x}, \bar{y})$ is given by $\bar{x} = \dfrac{M_y}{m}$ and $\bar{y} = \dfrac{M_x}{m}$, where $m = \rho \int_a^b [f(x) - g(x)]\,dx$ is the mass of the lamina.

EXAMPLE 4 The Center of Mass of a Planar Lamina

See LarsonCalculus.com for an interactive version of this type of example.

Find the center of mass of the lamina of uniform density ρ bounded by the graph of $f(x) = 4 - x^2$ and the x-axis.



The center of mass is the balancing point.

Figure 7.60

EXAMPLE 5 The Centroid of a Plane Region

Find the centroid of the region bounded by the graphs of $f(x) = 4 - x^2$ and $g(x) = x + 2$.

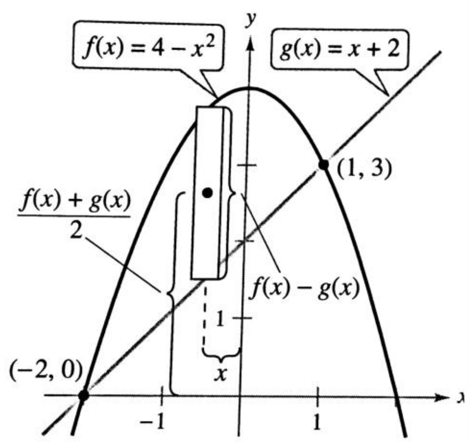
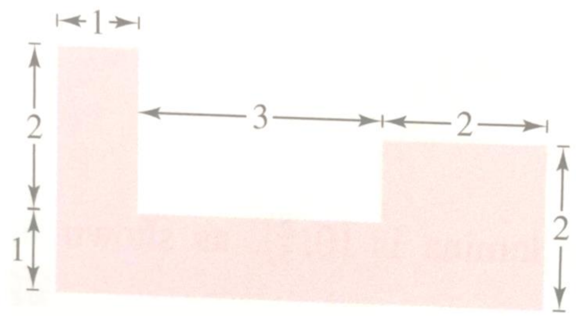


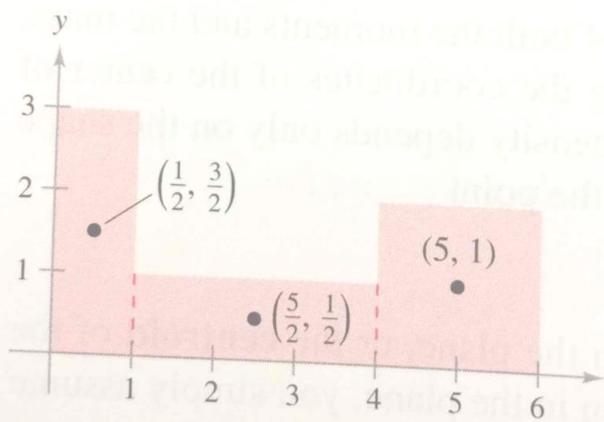
Figure 7.61

EXAMPLE 6 The Centroid of a Simple Plane Region

Find the centroid of the region shown in Figure 7.62(a).



(a) Original region



(b) The centroids of the three rectangles

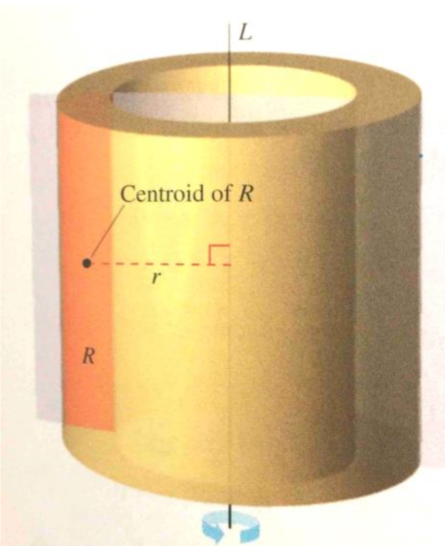
Figure 7.62

THEOREM 7.1 The Theorem of Pappus

Let R be a region in a plane and let L be a line in the same plane such that L does not intersect the interior of R, as shown in Figure 7.63. If r is the distance between the centroid of R and the line, then the volume V of the solid of revolution formed by revolving R about the line is

$$V = 2\pi rA$$

where A is the area of R. (Note that $2\pi r$ is the distance traveled by the centroid as the region is revolved about the line.)



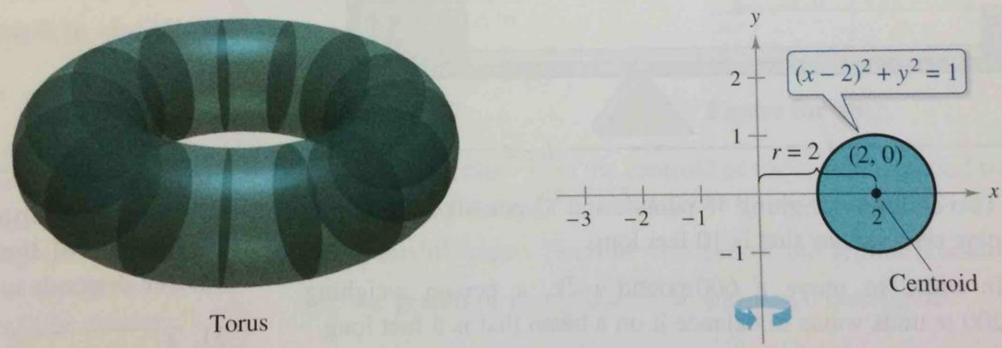
The volume V is $2\pi rA$, where A is th area of region R.
Figure 7.63

EXAMPLE 7 Finding Volume by the Theorem of Pappus

Find the volume of the torus shown in Figure 7.64(a), which was formed by revolving the circular region bounded by

$$(x - 2)^2 + y^2 = 1$$

about the y-axis, as shown in Figure 7.64(b).



(a) (b)

Figure 7.64