Right Triangle Definitions of Trigonometric Functions

See **Figure 4.30**. The six **trigonometric functions of the acute angle θ** are defined as follows:

$$\sin\theta = \frac{\text{length of side opposite angle }\theta}{\text{length of hypotenuse}} = \frac{a}{c} \qquad \csc\theta = \frac{\text{length of hypotenuse}}{\text{length of side opposite angle }\theta} = \frac{c}{a}$$

$$\cos\theta = \frac{\text{length of side adjacent to angle }\theta}{\text{length of hypotenuse}} = \frac{b}{c} \qquad \sec\theta = \frac{\text{length of hypotenuse}}{\text{length of side adjacent to angle }\theta} = \frac{c}{b}$$

$$\tan\theta = \frac{\text{length of side opposite angle }\theta}{\text{length of side adjacent to angle }\theta} = \frac{a}{b} \qquad \cot\theta = \frac{\text{length of side adjacent to angle }\theta}{\text{length of side opposite angle }\theta} = \frac{b}{a}$$

Table 4.2 Trigonometric Functions of Special Angles

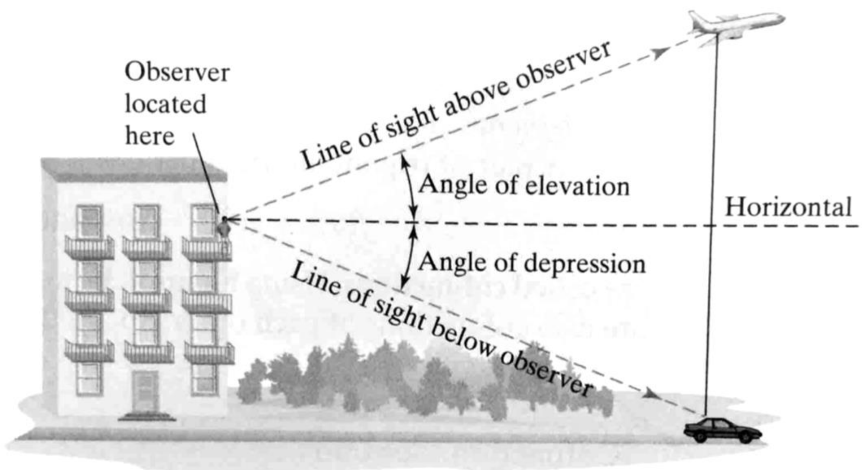
θ	$30° = \frac{\pi}{6}$	$45° = \frac{\pi}{4}$	$60° = \frac{\pi}{3}$
$\sin\theta$	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$
$\cos\theta$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$
$\tan\theta$	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$

Cofunction Identities

The value of a trigonometric function of θ is equal to the cofunction of the complement of θ. Cofunctions of complementary angles are equal.

$$\sin\theta = \cos(90° - \theta) \qquad \cos\theta = \sin(90° - \theta)$$
$$\tan\theta = \cot(90° - \theta) \qquad \cot\theta = \tan(90° - \theta)$$
$$\sec\theta = \csc(90° - \theta) \qquad \csc\theta = \sec(90° - \theta)$$

If θ is in radians, replace 90° with $\frac{\pi}{2}$.



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EXAMPLE 6 Problem Solving Using an Angle of Elevation

Sighting the top of a building, a surveyor measured the angle of elevation to be 22°. The transit is 5 feet above the ground and 300 feet from the building. Find the building's height.

Check Point 6 The irregular blue shape in **Figure 4.39** represents a lake. The distance across the lake, *a*, is unknown. To find this distance, a surveyor took the measurements shown in the figure. What is the distance across the lake? 333.9 yd

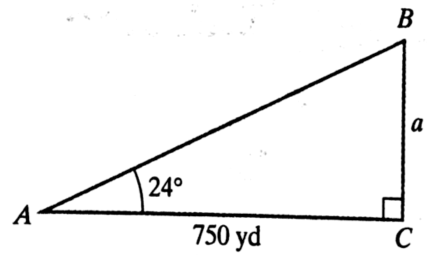


FIGURE 4.39

Example 8 Photography from a spy plane

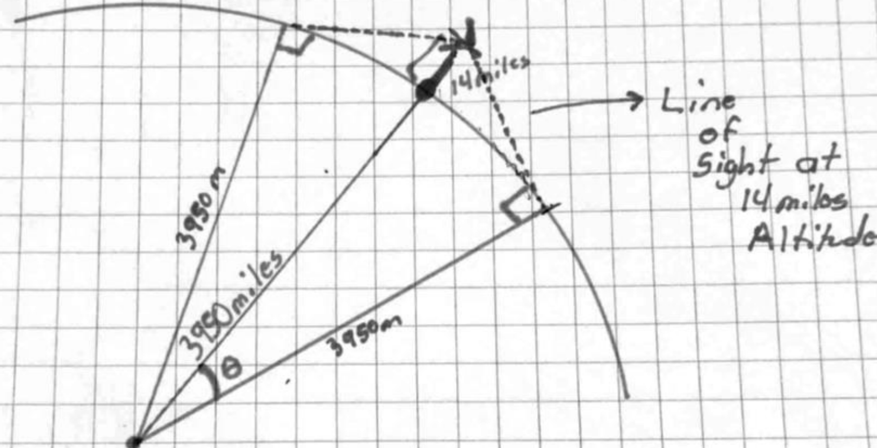
In the late 1950s, the Soviets labored to develop a missile that could stop the U-2 spy plane. On May 1, 1960, Nikita S. Khrushchev announced to the world that the Soviets had shot down Francis Gary Powers while Powers was photographing the Soviet Union from a U-2 at an altitude of 14 miles. How wide a path on the earth's surface could Powers see from that altitude? (Use 3950 miles as the earth's radius.)

EXAMPLE 7 Determining the Angle of Elevation

A building that is 21 meters tall casts a shadow 25 meters long. Find the angle of elevation of the sun to the nearest degree.

(6)

Example 8 Group Work (Spy Plane)

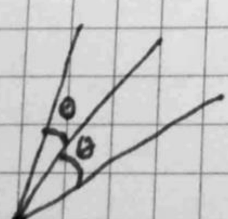


- The line of sight is a line tangent to a circle.
- Any tangential line to a cicle is ⊥ to the radius at the point of tangency.
- Radius = 3950
- Hypotenuse = 3950 + 14 = 3964
- We have adj and hypotenuse

$$\cos\theta = \frac{3950}{3964}$$

$$\theta = \cos^{-1}\left(\frac{3950}{3964}\right) \approx \boxed{4.8^\circ}$$

∴



$$\theta + \theta = 2\theta$$

$$\theta = 4.8^\circ$$

$$2(4.8^\circ) = \boxed{9.6^\circ}$$

(7)

- Thus, the width of the path seen is the length of the arc intercepted by the central angle 2θ (9.6°)
- Recall the formula for finding arc length is

$$S = \theta r \quad (\theta \text{ is in radians})$$

$$S = \frac{9.6^\circ}{1} \cdot \frac{\pi \text{ rad}}{180^\circ} \cdot \frac{3950 \text{ miles}}{1} =$$

$$\approx 661.8 \text{ miles}$$