Decomposition of $N(x)/D(x)$ into Partial Fractions

1. **Divide when improper:** When $N(x)/D(x)$ is an improper fraction (that is, when the degree of the numerator is greater than or equal to the degree of the denominator), divide the denominator into the numerator to obtain

$$\frac{N(x)}{D(x)} = (\text{a polynomial}) + \frac{N_1(x)}{D(x)}$$

where the degree of $N_1(x)$ is less than the degree of $D(x)$. Then apply Steps 2, 3, and 4 to the proper rational expression $N_1(x)/D(x)$.

2. **Factor denominator:** Completely factor the denominator into factors of the form

$$(px + q)^m \quad \text{and} \quad (ax^2 + bx + c)^n$$

where $ax^2 + bx + c$ is irreducible.

3. **Linear factors:** For each factor of the form $(px + q)^m$, the partial fraction decomposition must include the following sum of m fractions.

$$\frac{A_1}{(px + q)} + \frac{A_2}{(px + q)^2} + \cdots + \frac{A_m}{(px + q)^m}$$

4. **Quadratic factors:** For each factor of the form $(ax^2 + bx + c)^n$, the partial fraction decomposition must include the following sum of n fractions.

$$\frac{B_1x + C_1}{ax^2 + bx + c} + \frac{B_2x + C_2}{(ax^2 + bx + c)^2} + \cdots + \frac{B_nx + C_n}{(ax^2 + bx + c)^n}$$

EXAMPLE 1 **Distinct Linear Factors**

Write the partial fraction decomposition for

$$\frac{1}{x^2 - 5x + 6}.$$

EXAMPLE 2 **Repeated Linear Factors**

Find $\int \frac{5x^2 + 20x + 6}{x^3 + 2x^2 + x}\,dx.$

EXAMPLE 3 **Distinct Linear and Quadratic Factors**

See LarsonCalculus.com for an interactive version of this type of example.

Find $\int \frac{2x^3 - 4x - 8}{(x^2 - x)(x^2 + 4)}\,dx.$

EXAMPLE 4 **Repeated Quadratic Factors**

Find $\int \frac{8x^3 + 13x}{(x^2 + 2)^2}\,dx.$

GUIDELINES FOR SOLVING THE BASIC EQUATION

Linear Factors

1. Substitute the roots of the distinct linear factors in the basic equation.
2. For repeated linear factors, use the coefficients determined in the first guideline to rewrite the basic equation. Then substitute other convenient values of x and solve for the remaining coefficients.

Quadratic Factors

1. Expand the basic equation.
2. Collect terms according to powers of x.
3. Equate the coefficients of like powers to obtain a system of linear equations involving A, B, C, and so on.
4. Solve the system of linear equations.