Definition of Antiderivative

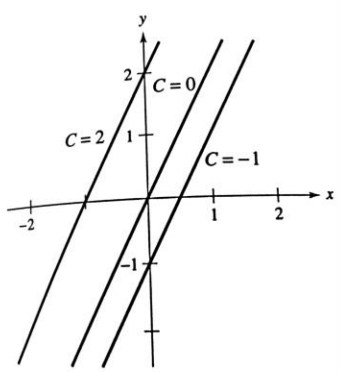
A function F is an **antiderivative** of f on an interval I when $F'(x) = f(x)$ for all x in I.

THEOREM 4.1 Representation of Antiderivatives

If F is an antiderivative of f on an interval I, then G is an antiderivative of f on the interval I if and only if G is of the form $G(x) = F(x) + C$, for all x in I where C is a constant.

EXAMPLE 1 Solving a Differential Equation

Find the general solution of the differential equation $y' = 2$.



Functions of the form $y = 2x + C$
Figure 4.1

Basic Integration Rules

Differentiation Formula	Integration Formula	
$\frac{d}{dx}[C] = 0$	$\int 0\,dx = C$	
$\frac{d}{dx}[kx] = k$	$\int k\,dx = kx + C$	
$\frac{d}{dx}[kf(x)] = kf'(x)$	$\int kf(x)\,dx = k\int f(x)\,dx$	
$\frac{d}{dx}[f(x) \pm g(x)] = f'(x) \pm g'(x)$	$\int [f(x) \pm g(x)]\,dx = \int f(x)\,dx \pm \int g(x)\,dx$	
$\frac{d}{dx}[x^n] = nx^{n-1}$	$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$	Power Rule
$\frac{d}{dx}[\sin x] = \cos x$	$\int \cos x\,dx = \sin x + C$	
$\frac{d}{dx}[\cos x] = -\sin x$	$\int \sin x\,dx = -\cos x + C$	
$\frac{d}{dx}[\tan x] = \sec^2 x$	$\int \sec^2 x\,dx = \tan x + C$	
$\frac{d}{dx}[\sec x] = \sec x \tan x$	$\int \sec x \tan x\,dx = \sec x + C$	
$\frac{d}{dx}[\cot x] = -\csc^2 x$	$\int \csc^2 x\,dx = -\cot x + C$	
$\frac{d}{dx}[\csc x] = -\csc x \cot x$	$\int \csc x \cot x\,dx = -\csc x + C$	

EXAMPLE 2 Describing Antiderivatives

$$\int 3x\,dx = 3\int x\,dx$$ Constant Multiple Rule

EXAMPLE 3

See LarsonCal

Original Integral

a. $\int \frac{1}{x^3}\,dx$

b. $\int \sqrt{x}\,dx$

c. $\int 2\sin x\,dx$

EXAMPLE 5 **Rewriting Before Integrating**

$$\int \frac{x+1}{\sqrt{x}}\,dx = \int \left(\frac{x}{\sqrt{x}} + \frac{1}{\sqrt{x}}\right) dx$$ Rewrite as tw

EXAMPLE 6 Rewriting Before Integrating

$$\int \frac{\sin x}{\cos^2 x}\,dx = \int \left(\frac{1}{\cos x}\right)\left(\frac{\sin x}{\cos x}\right)dx$$ Rewrite as a product.

EXAMPLE 7 R

Original Integral

a. $\int \frac{2}{\sqrt{x}}\,dx$

b. $\int (t^2 + 1)^2\,dt$

c. $\int \frac{x^3 + 3}{x^2}\,dx$

d. $\int \sqrt[3]{x}(x - 4)\,dx$

EXAMPLE 8 **Finding a Particular Solution**

Find the general solution of

$$F'(x) = \frac{1}{x^2}, \quad x > 0$$

and find the particular solution that satisfies the initial condition $F(1) = 0$.

EXAMPLE 9 Solving a Vertical Motion Problem

A ball is thrown upward with an initial velocity of 64 feet per second from an initial height of 80 feet.

a. Find the position function giving the height s as a function of the time t.

b. When does the ball hit the ground?

Solution

51. Tree Growth An evergreen nursery usually sells a certain type of shrub after 6 years of growth and shaping. The growth rate during those 6 years is approximated by $dh/dt = 1.5t + 5$, where t is the time in years and h is the height in centimeters. The seedlings are 12 centimeters tall when planted ($t = 0$).

(a) Find the height after t years.

(b) How tall are the shrubs when they are sold?

52. Population Growth The rate of growth dP/dt of a population of bacteria is proportional to the square root of t, where P is the population size and t is the time in days $(0 \leq t \leq 10)$. That is,

$$\frac{dP}{dt} = k\sqrt{t}.$$

The initial size of the population is 500. After 1 day the population has grown to 600. Estimate the population after 7 days.

60. Escape Velocity The minimum velocity required for an object to escape Earth's gravitational pull is obtained from the solution of the equation

$$\int v\,dv = -GM\int \frac{1}{y^2}\,dy$$

where v is the velocity of the object projected from Earth, y is the distance from the center of Earth, G is the gravitational constant, and M is the mass of Earth. Show that v and y are related by the equation

$$v^2 = v_0^2 + 2GM\left(\frac{1}{y} - \frac{1}{R}\right)$$

where v_0 is the initial velocity of the object and R is the radius of Earth.

Rewriting Before Integrating In Exercises 7–10, complete the table to find the indefinite integral.

	Original Integral	Rewrite	Integrate	Simplify
7.	$\int \sqrt[3]{x}\, dx$			
8.	$\int \frac{1}{4x^2}\, dx$			
9.	$\int \frac{1}{x\sqrt{x}}\, dx$			
10.	$\int \frac{1}{(3x)^2}\, dx$			

	Given	*Rewrite*	*Integrate*	*Simplify*
7.	$\int \sqrt[3]{x}\, dx$	$\int x^{1/3}\, dx$	$\frac{x^{4/3}}{4/3} + C$	$\frac{3}{4}x^{4/3} + C$
8.	$\int \frac{1}{4x^2}\, dx$	$\frac{1}{4}\int x^{-2}\, dx$	$\frac{1}{4}\frac{x^{-1}}{-1} + C$	$-\frac{1}{4x} + C$
9.	$\int \frac{1}{x\sqrt{x}}\, dx$	$\int x^{-3/2}\, dx$	$\frac{x^{-1/2}}{-1/2} + C$	$-\frac{2}{\sqrt{x}} + C$
10.	$\int \frac{1}{(3x)^2}\, dx$	$\frac{1}{9}\int x^{-2}\, dx$	$\frac{1}{9}\left(\frac{x^{-1}}{-1}\right) + C$	$\frac{-1}{9x} + C$