

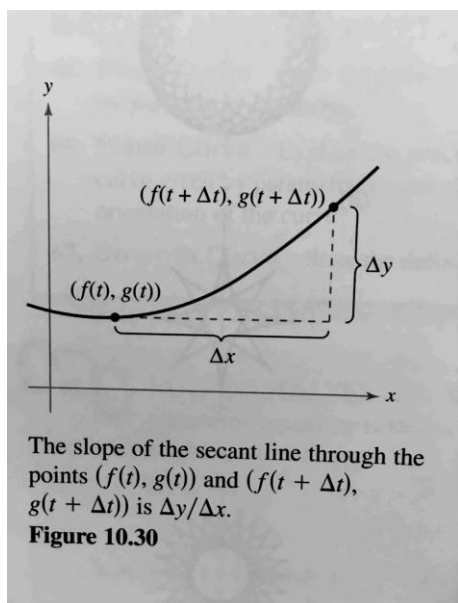
THEOREM 10.7 Parametric Form of the Derivative

If a smooth curve C is given by the equations

$$x = f(t) \quad \text{and} \quad y = g(t)$$

then the slope of C at (x, y) is

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}, \quad \frac{dx}{dt} \neq 0.$$



$$\frac{d^2y}{dx^2} = \frac{d}{dx} \left[\frac{dy}{dx} \right] = \frac{d}{dt} \left[\frac{dy}{dx} \right] \frac{dt}{dx}$$

Second derivative

$$\frac{d^3y}{dx^3} = \frac{d}{dx} \left[\frac{d^2y}{dx^2} \right] = \frac{d}{dt} \left[\frac{d^2y}{dx^2} \right] \frac{dt}{dx}$$

Third derivative

EXAMPLE 1**Differentiation and Parametric Form**

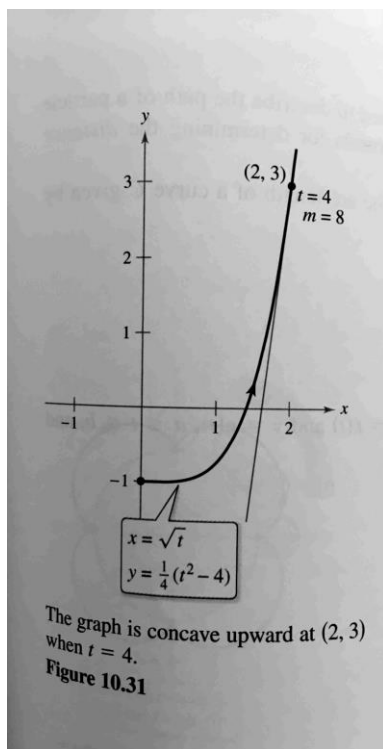
Find dy/dx for the curve given by $x = \sin t$ and $y = \cos t$.

EXAMPLE 2**Finding Slope and Concavity**

For the curve given by

$$x = \sqrt{t} \quad \text{and} \quad y = \frac{1}{4}(t^2 - 4), \quad t \geq 0$$

find the slope and concavity at the point $(2, 3)$.



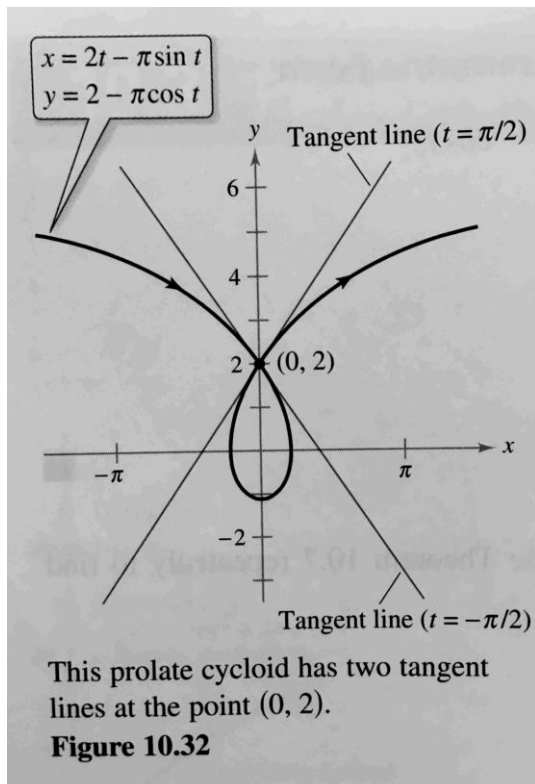
EXAMPLE 3**A Curve with Two Tangent Lines at a Point**

$t = \pi/2$) $\dots \blacktriangleright$ See *LarsonCalculus.com* for an interactive version of this type of example.

The **prolate cycloid** given by

$$x = 2t - \pi \sin t \quad \text{and} \quad y = 2 - \pi \cos t$$

crosses itself at the point $(0, 2)$, as shown in Figure 10.32. Find the equations of both tangent lines at this point.



THEOREM 10.8 Arc Length in Parametric Form

If a smooth curve C is given by $x = f(t)$ and $y = g(t)$ such that C does not intersect itself on the interval $a \leq t \leq b$ (except possibly at the endpoints), then the arc length of C over the interval is given by

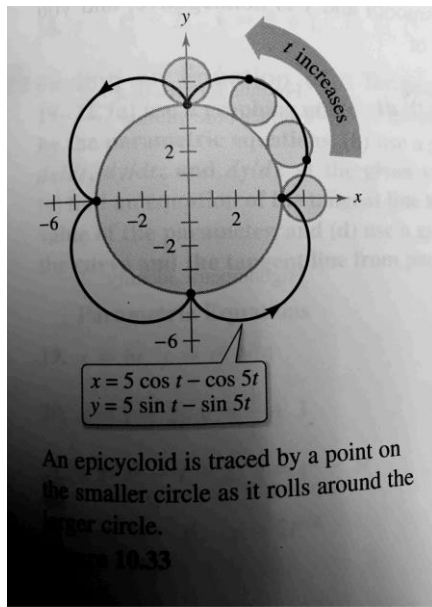
$$s = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt = \int_a^b \sqrt{[f'(t)]^2 + [g'(t)]^2} dt.$$

EXAMPLE 4 Finding Arc Length

A circle of radius 1 rolls around the circumference of a larger circle of radius 4, as shown in Figure 10.33. The epicycloid traced by a point on the circumference of the smaller circle is given by

$$x = 5 \cos t - \cos 5t \quad \text{and} \quad y = 5 \sin t - \sin 5t.$$

Find the distance traveled by the point in one complete trip about the larger circle.



THEOREM 10.9 Area of a Surface of Revolution

If a smooth curve C given by $x = f(t)$ and $y = g(t)$ does not cross itself on an interval $a \leq t \leq b$, then the area S of the surface of revolution formed by revolving C about the coordinate axes is given by the following.

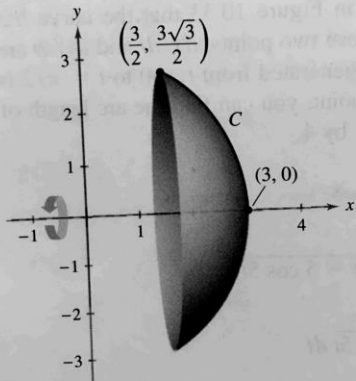
1. $S = 2\pi \int_a^b g(t) \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$ Revolution about the x -axis: $g(t) \geq 0$
2. $S = 2\pi \int_a^b f(t) \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$ Revolution about the y -axis: $f(t) \geq 0$

EXAMPLE 5 Finding the Area of a Surface of Revolution

Let C be the arc of the circle $x^2 + y^2 = 9$ from $(3, 0)$ to

$$\left(\frac{3}{2}, \frac{3\sqrt{3}}{2}\right)$$

as shown in Figure 10.34. Find the area of the surface formed by revolving C about the x -axis.



The surface of revolution has a surface area of 9π .

Figure 10.34

