THEOREM 4.9 The Fundamental Theorem of Calculus

If a function f is continuous on the closed interval $[a, b]$ and F is an antiderivative of f on the interval $[a, b]$, then

$$\int_a^b f(x)\,dx = F(b) - F(a).$$

GUIDELINES FOR USING THE FUNDAMENTAL THEOREM OF CALCULUS

1. *Provided you can find* an antiderivative of f, you now have a way to evaluate a definite integral without having to use the limit of a sum.
2. When applying the Fundamental Theorem of Calculus, the notation shown below is convenient.

$$\int_a^b f(x)\,dx = F(x)\Big]_a^b = F(b) - F(a)$$

For instance, to evaluate $\int_1^3 x^3\,dx$, you can write

$$\int_1^3 x^3\,dx = \frac{x^4}{4}\Big]_1^3 = \frac{3^4}{4} - \frac{1^4}{4} = \frac{81}{4} - \frac{1}{4} = 20.$$

3. It is not necessary to include a constant of integration C in the antiderivative.

$$\int_a^b f(x)\,dx = \Big[F(x) + C\Big]_a^b = [F(b) + C] - [F(a) + C] = F(b) - F(a)$$

EXAMPLE 1 Evaluating a Definite Integral

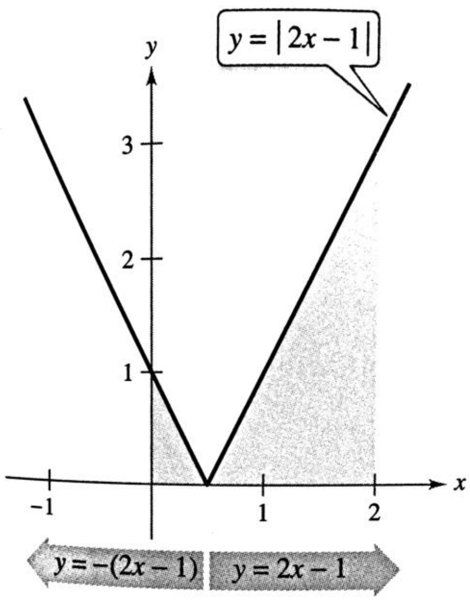
See LarsonCalculus.com for an interactive version of this type of exam

Evaluate each definite integral.

a. $\int_1^2 (x^2 - 3)\, dx$ **b.** $\int_1^4 3\sqrt{x}\, dx$ **c.** $\int_0^{\pi/4} \sec^2 x\, dx$

EXAMPLE 2 A Definite Integral Involving Absolute Value

Evaluate $\int_0^2 |2x - 1|\, dx$.



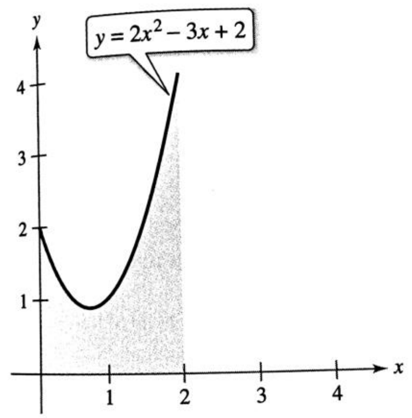
The definite integral of y on $[0, 2]$ is $\frac{5}{2}$.
Figure 4.28

EXAMPLE 3 **Using the Fundamental Theorem to Find Area**

Find the area of the region bounded by the graph of

$$y = 2x^2 - 3x + 2$$

the x-axis, and the vertical lines $x = 0$ and $x = 2$, as shown in Figure 4.29.



The area of the region bounded by the graph of y, the x-axis, $x = 0$, and $x = 2$ is $\frac{10}{3}$.
Figure 4.29

THEOREM 4.10 Mean Value Theorem for Integrals

If f is continuous on the closed interval $[a, b]$, then there exists a number c in the closed interval $[a, b]$ such that

$$\int_a^b f(x)\,dx = f(c)(b - a).$$

Definition of the Average Value of a Function on an Interval

If f is integrable on the closed interval $[a, b]$, then the **average value** of f on the interval is

$$\frac{1}{b - a}\int_a^b f(x)\,dx.$$

See Figure 4.32.

EXAMPLE 4 Finding the Average Value of a Function

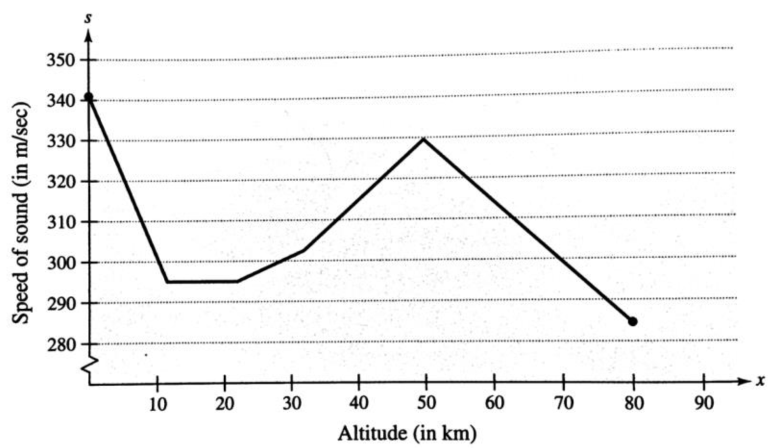
Find the average value of $f(x) = 3x^2 - 2x$ on the interval $[1, 4]$.

EXAMPLE 5 **The Speed of Sound**

At different altitudes in Earth's atmosphere, sound travels at different speeds. The speed of sound $s(x)$ (in meters per second) can be modeled by

$$s(x) = \begin{cases} -4x + 341, & 0 \le x < 11.5 \\ 295, & 11.5 \le x < 22 \\ \frac{3}{4}x + 278.5, & 22 \le x < 32 \\ \frac{3}{2}x + 254.5, & 32 \le x < 50 \\ -\frac{3}{2}x + 404.5, & 50 \le x \le 80 \end{cases}$$

where x is the altitude in kilometers (see Figure 4.34). What is the average speed of sound over the interval $[0, 80]$?

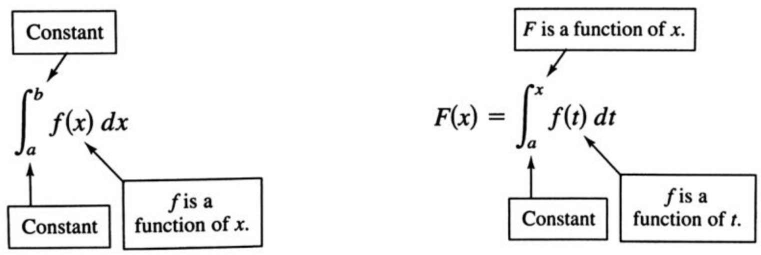


Speed of sound depends on altitude.
Figure 4.34

The Second Fundamental Theorem of Calculus

Earlier you saw that the definite integral of f on the interval $[a, b]$ was defined using the constant b as the upper limit of integration and x as the variable of integration. However, a slightly different situation may arise in which the variable x is used in the upper limit of integration. To avoid the confusion of using x in two different ways, t is temporarily used as the variable of integration. (Remember that the definite integral is *not* a function of its variable of integration.)

The Definite Integral as a Number **The Definite Integral as a Function of x**



EXAMPLE 6 The Definite Integral as a Function

Evaluate the function

$$F(x) = \int_0^x \cos t \, dt$$

at $x = 0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3},$ and $\frac{\pi}{2}$.

THEOREM 4.11 The Second Fundamental Theorem of Calculus

If f is continuous on an open interval I containing a, then, for every x in the interval,

$$\frac{d}{dx}\left[\int_a^x f(t)\,dt\right] = f(x).$$

EXAMPLE 7 The Second Fundamental Theorem of Calculus

Evaluate $\frac{d}{dx}\left[\int_0^x \sqrt{t^2+1}\,dt\right]$.

EXAMPLE 8 The Second Fundamental Theorem of Calculus

Find the derivative of $F(x) = \int_{\pi/2}^{x^3} \cos t\,dt$.

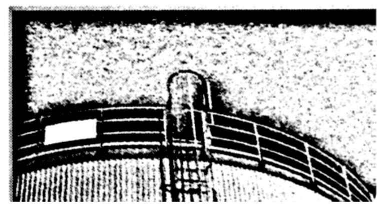
THEOREM 4.12 The Net Change Theorem

The definite integral of the rate of change of quantity $F'(x)$ gives the total change, or **net change,** in that quantity on the interval $[a, b]$.

$$\int_a^b F'(x)\,dx = F(b) - F(a) \qquad \text{Net change of } F$$

EXAMPLE 9 Using the Net Change Theorem

A chemical flows into a storage tank at a rate of $(180 + 3t)$ liters per minute, where t is the time in minutes and $0 \leq t \leq 60$. Find the amount of the chemical that flows into the tank during the first 20 minutes.



EXAMPLE 10 Solving a Particle Motion Problem

The velocity (in feet per second) of a particle moving along a line is

$$v(t) = t^3 - 10t^2 + 29t - 20$$

where t is the time in seconds.

a. What is the displacement of the particle on the time interval $1 \le t \le 5$?

b. What is the total distance traveled by the particle on the time interval $1 \le t \le 5$?