EXAMPLE 1 **Sketching**

Analyze and sketch the graph of

$$f(x) = \frac{2(x^2 - 9)}{x^2 - 4}.$$

Solution

First derivative:

Second derivative:

x-intercepts:

y-intercept:

Vertical asymptotes:

Horizontal asymptote:

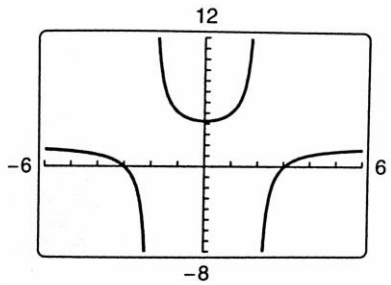
Critical number:

Possible points of inflection:

Domain:

Symmetry:

Test intervals:



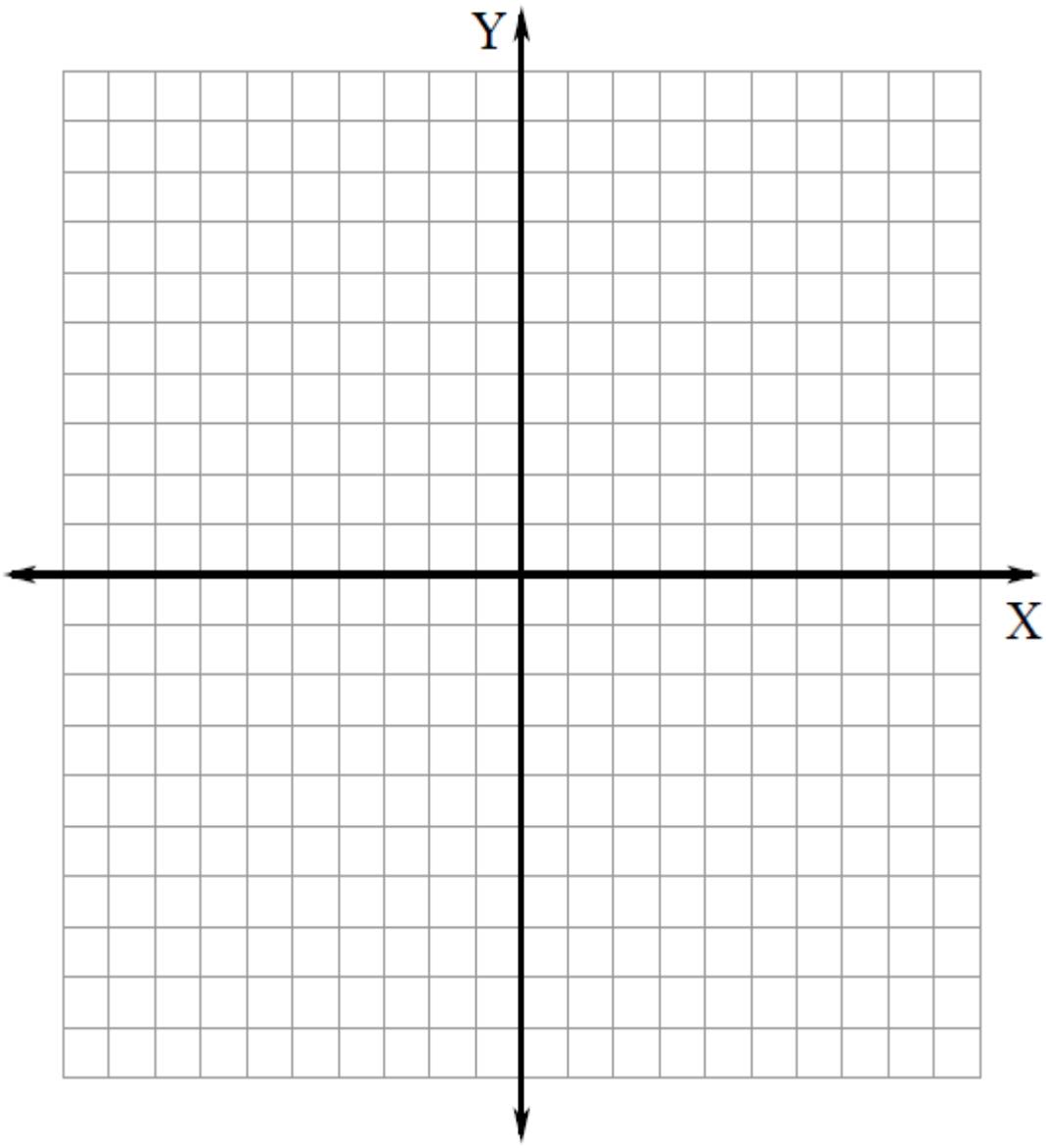
By not using calculus, you may overlook important characteristics of the graph of g.
Figure 3.46

EXAMPLE 2 **Sketching the Graph of a Rational Function**

Analyze and sketch the graph of $f(x) = \dfrac{x^2 - 2x + 4}{x - 2}$.

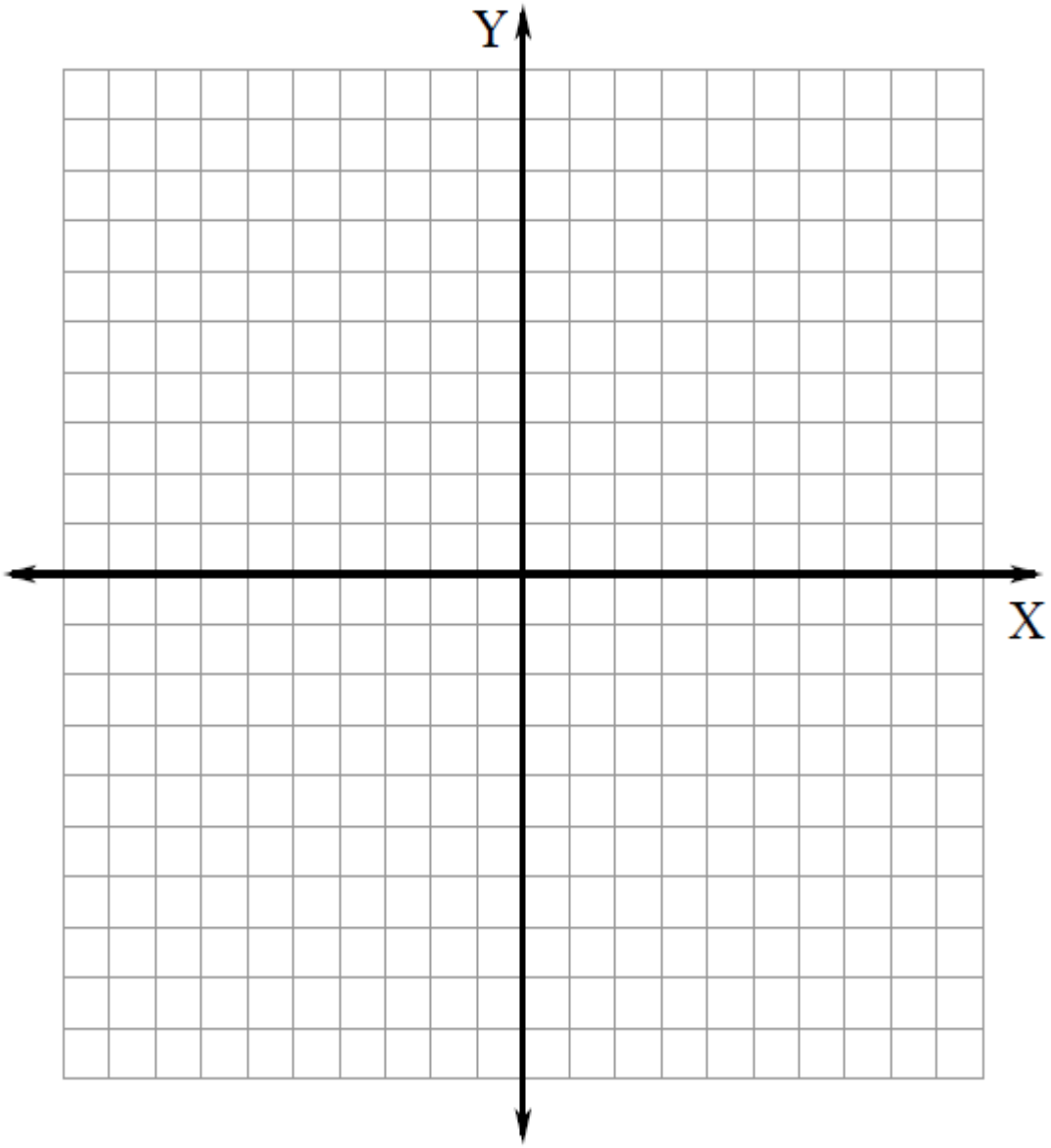
EXAMPLE 3 **Sketching the Graph of a Radical Function**

Analyze and sketch the graph of $f(x) = \dfrac{x}{\sqrt{x^2 + 2}}$.



EXAMPLE 4 **Sketching the Graph of a Radical Function**

Analyze and sketch the graph of $f(x) = 2x^{5/3} - 5x^{4/3}$.

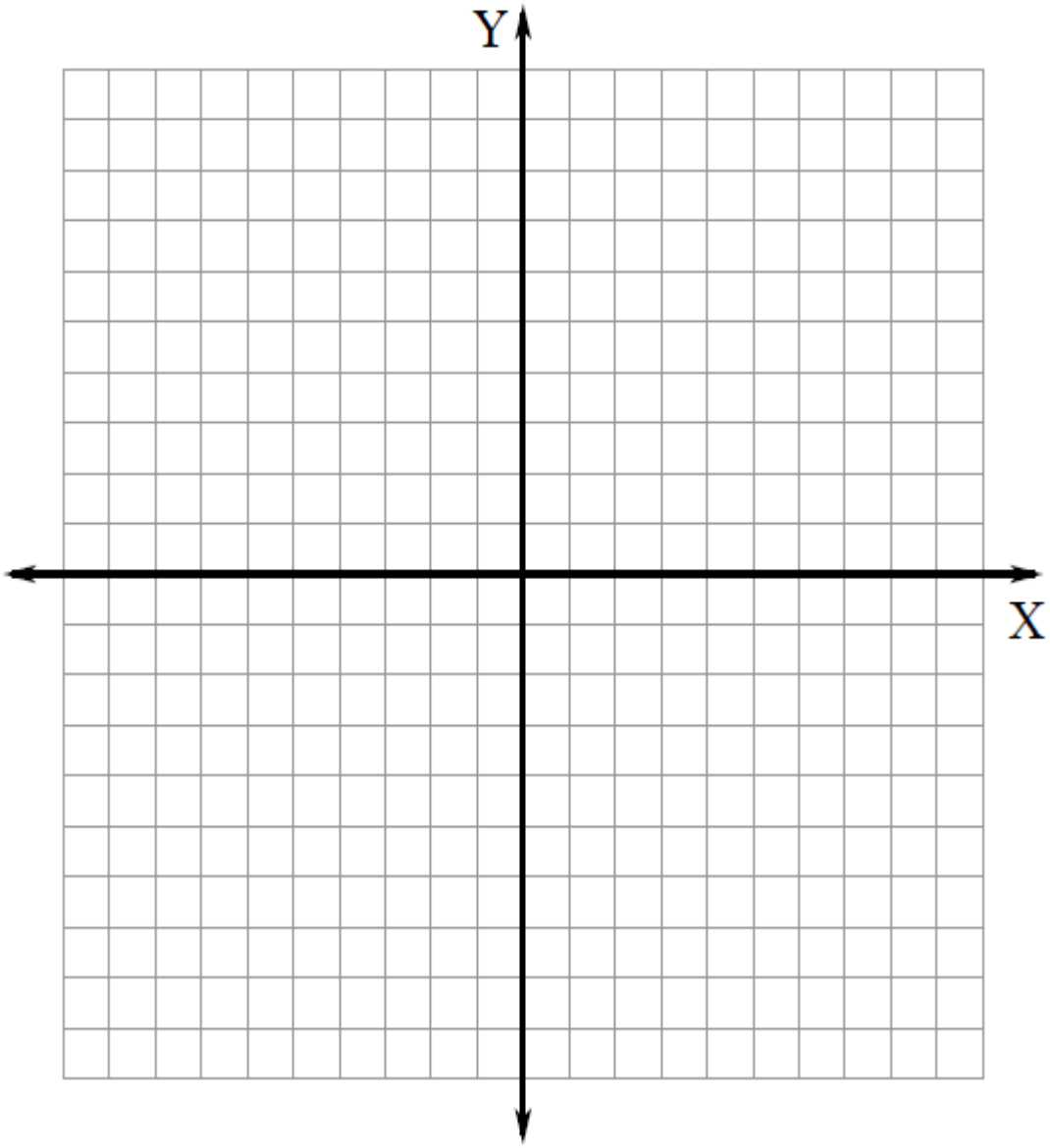


EXAMPLE 5 Sketching the Graph of a Polynomial Function

See LarsonCalculus.com for an interactive version of this type of example.

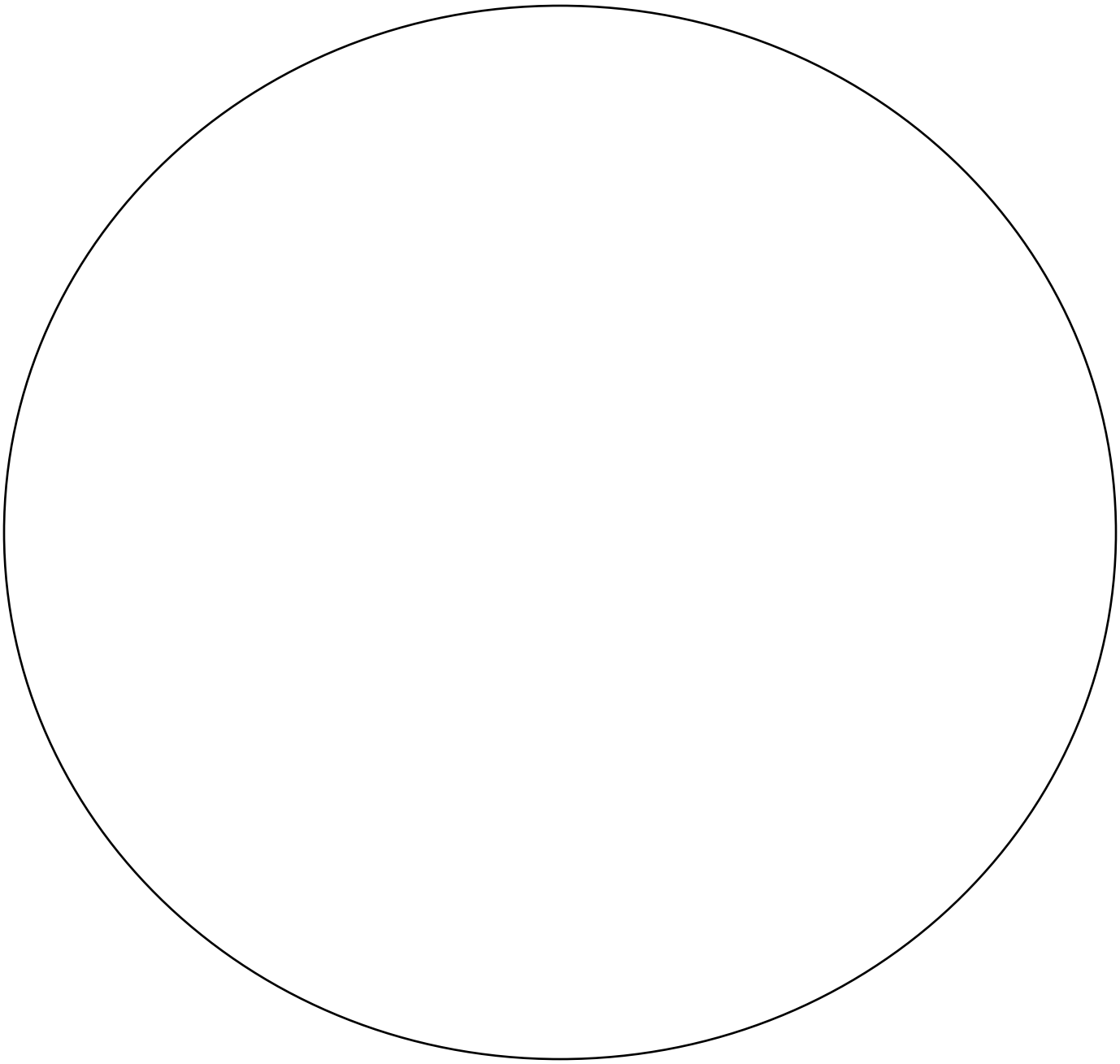
Analyze and sketch the graph of

$$f(x) = x^4 - 12x^3 + 48x^2 - 64x.$$

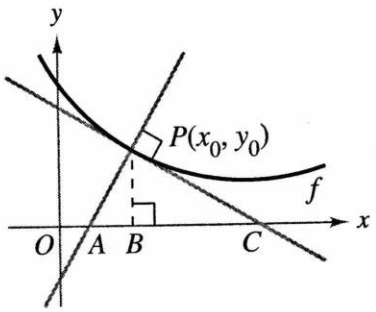


EXAMPLE 6 **Sketching the Graph of a Trigonometric Function**

Analyze and sketch the graph of $f(x) = (\cos x)/(1 + \sin x)$.



61. Investigation Let $P(x_0, y_0)$ be an arbitrary point on the graph of f such that $f'(x_0) \neq 0$, as shown in the figure. Verify each statement.



(a) The x-intercept of the tangent line is

$$\left(x_0 - \frac{f(x_0)}{f'(x_0)}, 0\right).$$

(b) The y-intercept of the tangent line is

$$(0, f(x_0) - x_0 f'(x_0)).$$

(c) The x-intercept of the normal line is

$$(x_0 + f(x_0)f'(x_0), 0).$$

(d) The y-intercept of the normal line is

$$\left(0, y_0 + \frac{x_0}{f'(x_0)}\right).$$

(e) $|BC| = \left|\dfrac{f(x_0)}{f'(x_0)}\right|$

(f) $|PC| = \left|\dfrac{f(x_0)\sqrt{1 + [f'(x_0)]^2}}{f'(x_0)}\right|$

(g) $|AB| = |f(x_0)f'(x_0)|$

(h) $|AP| = |f(x_0)|\sqrt{1 + [f'(x_0)]^2}$